University of Khartoum
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Yield Line Method Applied to Slabs with Different Supports

Thesis Submitted in Partial Fulfilment of requirements for the degree of M.sc. in Structural Engineering

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Dedication

To my mother, who is the first teacher.

To the soul of my father, who earned my living and my needs.

To all my brothers & sisters.

Acknowledgement

My greatly indebted to professor E.I. ElNiema for his guidance and encouragement and for his invaluable assistance.

I wish to record also my profound thanks to Eng. Sami Mahmoud Eltoum And Mr. Khalid Dongula for their invaluable hel, and to Civil Eng. Department of Khartoum University for the facilities offered to carry out this research.

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هذا البحث

في العهود السابقة بل وحتى وقت قريب لـم تكـن الخرسـانة بمكوناتهـا (الأسمنت، الركام، الرمل) معروفة لدى المهتمين بعمليات البناء والتشييد وكانـت معظم الإنشاءات والمباني تشيد من الأحجار والطين والحديد والخشب لأنها المواد المتوفرة آنذاك، ولكن وفي تطور مذهل أصبحت الخرسانة وفور معرفتها المـادة الأهم، تعتبر أكثر من 65% من الإنشاءات في العالم اليوم مصنعة من الخرسانة.

ولما كانت كمية الخرسانة المستخدمة في البلاطات الخرسـانية المـسلحة المسطحة أكبر من كمية الخرسانة المستخدمة فـي القواعـد والأعمـدة والأبيـام وغيرها...، لأنها عادةً ما تشغل مساحات واسعة بسمك مقدر، كما أنها تعد مـن أهم أسباب تشوه المنشآت والمباني الخرسانية وانهياراتها ما لم تصمم بمواصفات عالية الدقة والجودة خاصةً في المباني الضخمة.

اهتم هذا البحث بدراسة البلاطات الخرسانية المسطحة بحدودها المختلفـة وتحليلها لمعرفة القوى المؤثرة عليها ومعرفة قوى تحملها ومن ثم تصميمها.

توجد طرق عديدة لتحديد القوى الواقعة على البلاطات المسلحة إلا أنه وفي هذا البحث استخدمت طريقة خطوط الخضوع (Yield Lines).

(التشققات) كطريقة سهلة ومثلى ومن ثم تفصيلها بطريقتي العمل الـوهمي والاتزان (Virtual Work and Equilibrium Method).

من ناحية علمية يتلخص هذا البحث بصورة أساسـية لتحليـل البلاطـات الخرسانية المسطحة مع الإشارة أن هذا التسطيح يعطـي البلاطـات الخرسـانية المثبتة جانبياً قوى تحمل إضافية للأحمال المساطة عليها تعرف هذه القوى بقـوة التسطح (Membrance Action)، هذه القوة تمت دراستها من خلال دراسات عديـدة، قدمنا في هذا البحث تلخيص لتلك الدراسات.

وقد تم تدعيم هذه الطرق النظرية بصب نماذج صغيرة لبلاطات مختلفة في التثبيت وتوزيع حديد التسليح ومن ثم تم تعريضها لأحمال تزيد تـدريجياً حتـى مرحلة الخضوع والانهيار ومن خلال التجارب تم رصد شكل التـشققات وقـوى

التحمل والهبوط ومقارنتها مع الحل النظري. فوجدنا أن النتائج آمنة وبعامل أمان مرضي.

نوصي باستخدام طريقة خطوط الخضوع عند تحليل البلاطات المسطحة ومع الأخذ في الاعتبار قوة التسطيح الأفقية لأن ذلك يؤدي وبصورة أساسية لتقليل حجم الخرسانة المستخدمة في المنشآت وبالتالي تقليل السمك وكمية وتكلفة المواد الخرسانية المستخدمة.

Abstract

In very ancient decades and up to a rather recent time, concrete in its known composites; cement, aggregate and sand, was not a well-known material to those of major concern of building construction field. Nevertheless, most of the formally constructed buildings were basically made of woods, stones, clay, steel and the like of the available materials at the time later and in an outrageous development, concrete has become, in no noticeable time, the most important material in building construction field. Nowadays, more than 65% of the structures in the world are made of concrete.

This research aims to study reinforced concrete slabs in different boundary conditions by analyzing the applied forces in order to determine reason why this research deals only with reinforced concrete slabs is that the quantity of concrete used in constructing them is more than that used in foundations, columns, beams and other reinforced concrete members. From the other hand, slabs, at most times, unless designed properly, are the main cause of failure of reinforced concrete structures.

There are multiple procedures to determine the forces applied on slabs. The Yield Line theory which is an ultimate load analysis, is used here as one easy and ideal method of analysis based on Virtual Work and Equilibrium Methods. Nevertheless, one should have in mind that when the slabs are restrained against lateral movement by stiff boundary elements compressive forces are induced in the plane of slab causing a rather considerable increase in the slab strength. These induced forces are known as membrane action and are studied here in the light of former studies.

Further in this research, the features of cracks, ultimate loads and deflection values are obtained theoretically using reinforced concrete slab models differ in fixing conditions and steel reinforcement densities, applied to loads increasing gradually until failure. The experimental results are then compared with the theoretical results obtained sooner in the research and the factor of safety is satisfactory.

Thus, we recommend the yield line method of analysis to be used taking into consideration the membrane\e action as it leads to reduce the volume of concrete by reducing the thickness of the slab and consequently reduce the cost of the structure.

Notation

Symbols	Description	Units
A_s	Reinforcement area	mm^2
A_{sb}	Reinforcement area in bottom side	mm^2
A_{st}	Reinforcement area in top side	mm^2
b	Slab width	mm
C	Compression force of concrete	-
D_1, d_2	Effective depths	mm
E_c	Modulus of elasticity of concrete	N/mm^2
E_s	Modulus of elasticity of steel	N/mm^2
e_c	Ultimate strain of concrete	-
e_y	Yield strain of steel	-
f_c	Crushing strength of concrete cylinder	N/mm^2
f_{cu}	Compressive strength of concrete	N/mm^2
f_y	Yield stress of reinforcement	N/mm^2
h	Overall slab thickness	mm
m, m'	Resisting moments along yield lines/unit length	kN.m/m
m_{um}	Ultimate moment of resistance per unit length	kN.m/m
m_x	Bending moment along x-axis	kN.m/m
m_y	Bending moment along y-axis	kN.m/m
P_u	Ultimate load	kN
S	Depth of stress block	Mm
T	Tension force of steel	kN
W_{ex}	External work	mm
W_{in}	Internal work	kN
X	Depth of neutral axis	kN.m
Z	Lever arm	kN.m

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Chapter One

Introduction & Literature Review to Yield – line analysis of slab

1.1 Introduction:

There are many possible approaches to the analysis and design of reinforced concrete slab systems. The various approaches available are elastic theory, limit analysis theory, and modification to elastic theory. Such methods can be used to analyse a given slab system to determine either the stresses in the slab and the supporting system or the load carrying capacity. Alternatively, the methods can be used to determine the distribution of moments and shears to allow the reinforcing steel and concrete sections to be designed.

The bending and torsional moments, shear forces, and deflections of slab system, with given dimensions, steel content, and material properties, at any stage of loading from zero to ultimate load, can be determined analytically using the conditions of static equilibrium and geometrical compatibility if the moment deformation relationships of the slab elements, and the yield criteria for bending and torsional moments and shear force, are known.

In such analysis to the complete behavior of slab systems, difficulties are caused by the nonlinearity of the high levels of stress. At low levels of loads the slab elements are uncracked and the action and deformations can be computed from elastic theory using the uncracked flexural rigidity of the slab elements.

1.2 Literature review: (3)

1.2.1 Basis of yield line theory:

The method for the limit state analysis of reinforced concrete slabs known as yield line theory was initiated by Ingerslev and greatly extended and advanced by Johasen. This method is an upper bound approach.

The ultimate load of the slab system is estimated by postulating a collapse mechanism which is compatible with the boundary conditions. The moments at the plastic hinge lines are the ultimate moments of resistance of the sections, and the ultimate load is determined using the principle of virtual work or the equations of equilibrium.

Being an upper bound approach the method gives an ultimate load for a given slab which is either correct or too high. The regions of the slab between the lines of plastic hinges are not examined to ensure that the moments there do not exceed the ultimate moments of resistance of the sections. But the ultimate moments of resistance between the lines of plastic hinges will only be exceeded if an in correct collapse mechanism is used. Thus, all the possible collapse mechanism of the slab must be examined to ensure that the load-carrrying capacity of the slab is not over estimated. It is to be noted that yield line theory assumes a flexural collapse mode, that is, that the slab has sufficient shear strength to prevent a shear failure.

The early literature on yield line theory was mainly in Danish and in 1953 Hognestad[(3)] produced the first summary of this work in English. Extensively treated in publications by Wood, Jones[(3)], the European concrete committee and others. This chapter will summarize the theory and enable the ultimate load of arrange of slabs with known boundary conditions and type of loading to be determined.

1.2.2 Condition of Ultimate Load[(1)]:

When a simply supported istropically reinforced square slab is subjected to a uniformly distributed load of increasing intensity, Initially we observe that the slab behaves elastically. As the load is gradually increased, cracking of the concrete on the tension side of the slab will cause the stiffness of the cracked section to be reduced, and the distribution of moments in the slab to change slightly. Owing to this redistribution, for equal Increments of load, the increase in moment at an uncracked section will be grater than before cracking occurred.

As the load is increased further – the reinforcement will yield in the central area of the slab, which is the region of highest moment. Once the steel in an under – reinforced section has yielded, although the section will continue to deform, its resistance moment will not increase by any appreciable amount, and consequently an even greater redistribution of moment takes place.

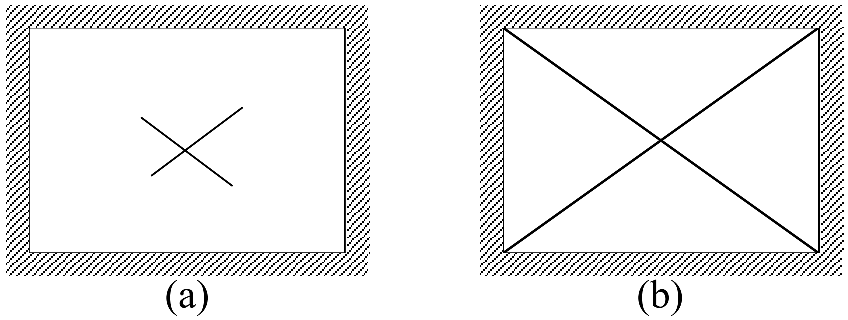


Fig (1.01)

When even more load is applied, since an increased proportion of moment has to be carried by the sections adjacent to the central area, this will cause the steel in the sections to yield as well. In this manner, lines along which the steel has yielded are propagated from the point at which yielding originally occurred. At this a stage of loading the yield lines might be as shown in figure (1.01 a), the application of more load will cause the reinforcement in even more sections to yield and further propagation of the yield lines, until eventually all the yield lines reach the boundary of the slab. This is shown in figure (1.01 b) at this stage since the resistance moments along the yield lines are almost at their ultimate values, and since the yield lines can not propagate further, the slab is carrying the maximum load, possible any slight increase in load will now cause a state of unstable equilibrium and the slab will continue to deflect under this load until the curvature at some section along the yield lines is so great that the concrete will crush. This section will then lose its capacity to carry any moment and this will increase the state of unstable equilibrium even more and cause failure to occur along the whole length of the yield lines. Thus the condition when the yield lines have just reached, the boundary of the slab may be regarded as the collapse criterion of the slab. The system of yield lines or fracture lines such as that in figure (1.01b) is called a yield – line pattern.

The first stage of the ultimate load analysis of any slab is to predict the yield – line pattern at failure. For given amount of reinforcement we can calculate the ultimate resistance moment along the yield lines, and hence by analyzing the slab at the failure condition we can find the value of the load which is in equilibrium with these moments.

As with most methods of analysis certain assumptions are made, which from tests are known to be reasonably true. Since the steel has yielded along the yield lines, the curvature of the slab in this region is larger than the curvature of the parts of the slab between the yield lines, which are still behaving elastically. Consequently is quite reasonable to assume that the elastic deformations are negligible in comparison with the palsaic deformation, in other words the assumption is made that the elements between the yield lines remain plane, and that all the deformations take place in the yield lines.

Thus in the deflected state, the plane elements of a slab such as A, B, C and D in figure (1.02) are inclined planes. Since the intersections between inclined planes are straight lines, it follows that the yield lines, which are the intersection between the plane elements, are also straight.

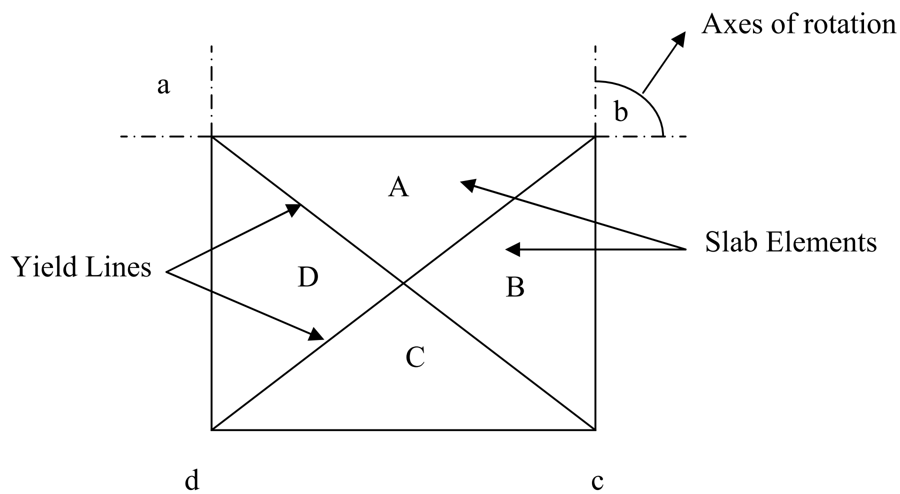


Figure (1.02)

In order that the slab may deflect, the element must rotate about certain axes of rotation in figure (1.02) element A rotates about ab, and element B about bc.

1.2.3 Main Assumptions:- (1) & (3)

The previous statement concerning yield lines may be summarized into four conditions, which help to predict the yield – line pattern for any slab.

A. Yield lines are straight.

B. Yield lines end at a slab boundary.

C. A yield line or yield line produced, passes through the intersection of the axes of rotations of adjacent slab elements.

D. Axes of rotation generally lie along lines of supports and pass over any columns.

1.2.4 Sign of Yield Lines:- (1)&(8)

In order to represent in a diagrammatic form the boundary conditions of any slab and the sign of the yield lines the rotation given in figure (1.03) will always be adopted.

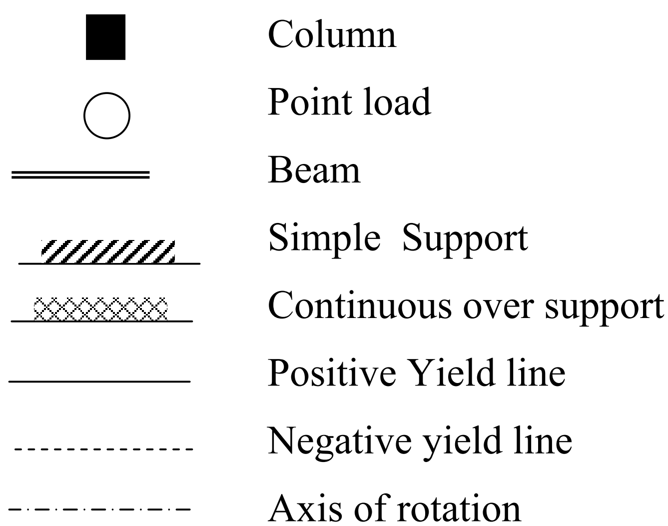


Fig (1.03)

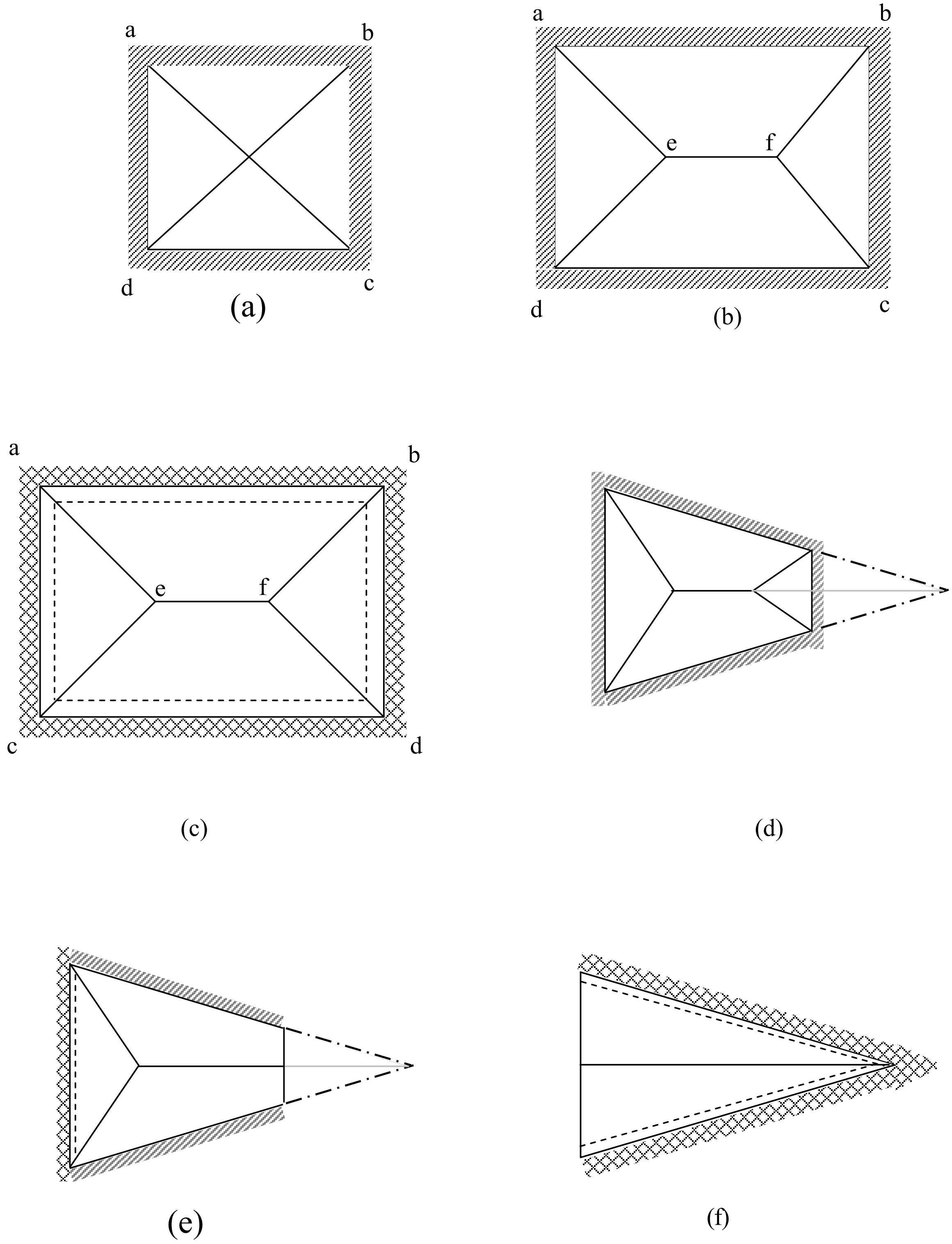
1.2.5 Postulate of the yield line pattern:- (1)&(3)

Since the first step in any analysis is to postulate a failure mechanism or yield Line pattern, the yield line patterns of various slabs are shown in figure (1.04a – 1.04k) to show a possible yield line pattern and four conditions are conform. Figure (1.04a) shows a possible yield line pattern for square slab subjected to uniformly distributed load. The axes of rotation of element A is ab, the line of support while that of B is bc. The yield line between these elements passes through the point of intersection of these axes, which is the corner b. Similarly yield lines pass through the other corners.

Since yielding starts in the center of slab, then the yield lines are straight lines between the center and the corners. Figure (1.04 b) shows the yield pattern for rectangular slab under uniformal load. The yield lines pass through the corners for the reasons given previously, and yield line (ef) parallel to the longer sides – it intersects the parallels axes of rotation of adjacent element A and C at infinity. Initially it is only necessary to draw the general yield – line pattern, the exact position of (e) and (f) can be found in the process of analysis.

For fixed edges slab – continuous – rectangular slab shown in figure (1.04 c) negative yield lines must also form along the line of support before they can become axes of rotation.

Other patterns shown in figure (1.04b – 1.04k) may be found by similar reasoning.



a
b
d
c
(a)
a
b
e
f
d
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(b)
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b
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(d)
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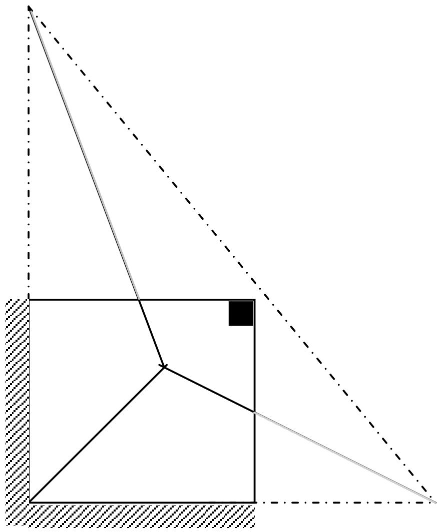
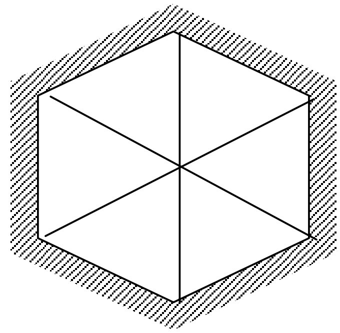
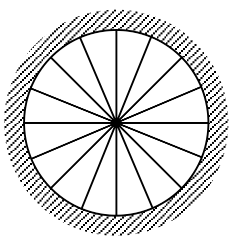
(f)

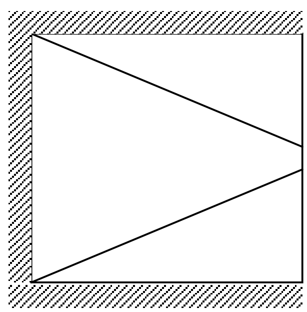




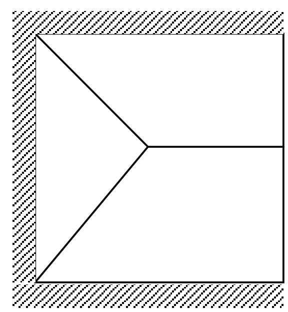
h

(g)





or



(j)

(k)

Two possible yield line pattern

Figure (1.04)

1.2.6 Methods of Solution:

In order to find the relation between the ultimate resistance moments in the slab and the ultimate load. Two methods of solution are available:

(1) Yield – line analysis by virtual work.

(2) Yield – line analysis by Equilibrium.

The aim of this research is to discussed these two methods.

1.3 Upper and Lower Bound Theorems:

Plastic analysis methods such as yield line theory derived form the general theory of structural plasticity, which states that the ultimate collapse load of a structure lies between two limits, an upper bound and lower bound of the true collapse load. These limits can be found by well-established methods. A full solution by the theory of plasticity would attempt to make the lower and upper bounds converge to a single correct solution.

The lower bound theorem and upper bound theorem when applied to the slabs can be stated as follows:

1.3.1 Lower bound theorem:

If, for a given external load, it is possible to find distribution of moments that satisfies equilibrium requirements, with the moment not exceeding the yield moment at any location, and if the boundary conditions are satisfied, then the given load is a lower bound of the true carrying capacity. Lower Bound Theorem (Sometimes called the static theorem). The theorem is often referred to as the safe theorem.

1.3.2 Upper bound theorem:

If, for a small increment of displacement, the internal work done by slab, assuming that the moment at every plastic hinge is equal to the yield moment and the boundary conditions are satisfied, is equal to the external work done by the given load for the increment of displacement, then that load is an upper bound of the carrying capacity. Upper bound theorem (sometimes called the kinematic theorem). The upper bound theorem is often referred as unsafe theorem, because interpreted in a design sense, it states that the value of the plastic moment obtained on the basis of an arbitrarily assumed collapse mechanism is smaller than, or at best equal to, that actually required.

If the lower bound conditions are satisfied, the slab can certainly carry the given load, although a higher load may be carried if internal redistributions of moment occur. If the upper boundry conditions are satisfied, a load greater than the given load will certainly cause failure, load may be carried if internal redistributions of moment occur. If the upper boundry conditions are satisfied, a load greater than the given load will certainly cause failure, although a lower load may produce collapse if selected failure mechanism is incorrect in any sense.

In practice, in the plastic analysis of structures, one works either with the lower bound theorem or the upper bound theorem not both, and precautions are taken to ensure that the predicted failure load at least closely approaches the correct value.

Yield line theory gives upperbound solution results that are either correct or theoretically unsafe see table 1.1 .However, once the possible failure pattern that forms have been recognized, it is difficult to get the yield line analysis critically wrong. The mention of 'unsafe' can put designers off, and upper bound theories are often denigrated. However, if any results is out by small amount it can be regarded as theoretically unsafe.

Yet few practicing engineers regarded any analysis as being an absolutely accurate and make due allowance in their design. The same is true and acknowledged in practical yield line design.

Table 1.1 Upper and Lower Bound Ultimate Load Theories:

Ultimate load theories for slabs fall into the following categories:

- Upperbound (unsafe or correct) or
- Lowerbound (safe or correct).

Plastic analysis is either base on:

- Upperbound (kinematic) methods, or on
- Lowerbound (static) methods.

Upperbound (kinematic) methods include:

- Plastic or yield line method for beams, frames.
- Yield line theory for slab.

Lowerbound (static) methods include:

- strip method for slabs,
- the strut and tie approach for deep beams, corbel, anchorages, walls and plates loaded in their plane.

In the majority of cases encountered, the result of yield line analysis from first principles will be well within 10% of the mathematically correct solution. The pragmatic approach, therefore, is to increase moment (or reinforcement) derived from calculation by 10%.

There are other factors that make yield line design safer than it may at the first appear, e.g. compressive membrane action in failing slabs (this alone can quadruple ultimate capacities), strain hardening of reinforcement, and the practice of rounding up steel areas when allotting bars to designed areas of steel required.

The practical designer can use yield theory with confidence, in knowledge that he or she is in control of a very useful, powerful and reliable design tool.

1.4 Serviceability and Deflection:

Yield line theory concerns itself with the ultimate Limit State. [The designer must ensure that relevant serviceability requirement; particularly the limits of deflection are satisfied.]

Deflection of slabs should be considered on the basis of elastic design. This may call for separate analysis but, more usually, deflection may be checked by using span/effective depth ratios with ultimate (i.e. yield line) moments as the basis. Such checks will be adequate in the vast majority of cases.

1.4.1 BS 8110:

Deflection usually ckecked by ensuring that allowable span/effective ratio is greater than the actual span/effective ratio (or by checking allowable span is greater than actual span). The basic span/effective ratio is modified by factors for compression reinforcement (if any) and service stress in the tension reinforcement. The latter can have a large effect when determining the service stress, f_s, to use in equation 8 in Table 3.10 of BS8110. When calculations are based on the ultimate yield line moments, one can conservatively, use β_b values of 1.1 for end spans and 1.2 for internal spans.

Where estimates of actual deflections are required, other approaches, such as the rigorous methods in BS 8110 part2, simplified methods or finite element methods should be investigated. These should be carried out as a secondary checked after the flexural design based on ultimate limit state principles has been carried out.

In order to keep cracking to an acceptable level it is normal to comply (sensibly) with the bar spacing requirements of BS 8110 clauses 3.12.11.2.7 and 2.8.

1.4.2 Eurocode2

Eurocode treats deflection in a similar manner to BS 8110. Deemed to satisfy span to depth ratios may be used to checked deflection. Otherwise calculations, which recognize that sections exist in a state between uncracked and fully cracked, should be undertaken.

1.5 Membrance Action in Slabs

The yield line theory due to virtual work and equilibrium the presence of only moments and shear forces at yield lines in the slab and gives a good indications of ultimate load when the yield line pattern can form without the development of membrance (in-plane) forces in the slab.

However, membrance action forces are often present in reinforced concrete slabs at the ultimate load as a result of boundary conditions and the geometry of deformation of the slab segments.

If the edges of slabs are restrained against lateral movement by stiff boundry elements, compressive membrance forces are induced in the plane of slab when, as the slab deflects, changes of geometry cause the slab edges to tend to move outward and react against the bounding elements. The compressive membrance forces so induced enhance the flexural strength of the slab sections at the yield lines (provided that the slab is not overreinforced, which will case the ultimate load of the slab to be larger than the ultimate load calculated using virtual and equilibrium methods at larger deflections the slab edges tend to move in ward and if the edges are laterally restrained, tensile membrance forces are induced, which may enable the slab to carry significant load by catenory action of the reinforcing steel.

Even in slabs that are not intentionally restrained against lateral movement at the edges, the deflection of the slab at the ultimate load may result in a more favorable distribution of internal forces in the slab, which can enhance carrying capacity as was indicated in the – the load experimental studies of reinforced concrete slabs discussed at the chapters (2) & (3).

A number of research studies on membrane action in reinforced concrete slabs have been conducted. However, only approximate ultimate strength thearies have been developed at present and the studies have pointed to difficulties in incrorporating membrance action in design.

Neverthe less, there is no doubt that membrane action will increase the ultimate load of many reinforced concrete slab systems significantly, even if membrane action has not been considered in the design.

Chapter Two
Yield – Line Analysis by Virtual Work

2.1 Introduction[2] :-

Since the moment and the load are in equilibrium when the yield – line pattern has formed, the slightest increment in load will cause the structure to deflect. When this increase in load Is infinitesimal the work done on the slab while the yield line are rotating must be equal to the loss of work due to the load deflecting.

Thus, if a point on the slab is given a virtual deflxion takes place along the yield lines. The internal work done on the slab will be the sum of the rotations in the yield lines multiplied be resisting ultimate moments. While the external loss of work will be the sum of the loads multiplied by their respective deflections. When the internal and external work is equated, we have the relation between the ultimate resistance moments in the slab and the ultimate load. This methods of solution is well known as principle of virtual work.

Usually we express the ultimate bending strength in terms of moment per unit width of slab, and conventional method of indicating the ultimate flexural strength is shown fig. 2.01. The side key indicates that the ultimate moment / unit length along a positive yield line in direction be is m, and the ultimate moment

/ unit length along a positive yield line in the direction ab is μm[1].

This implies of course that steel to produce the moment m is at right angles to this direction, that is in the direction ab, while the steel to produce the moment μm in the direction bc.

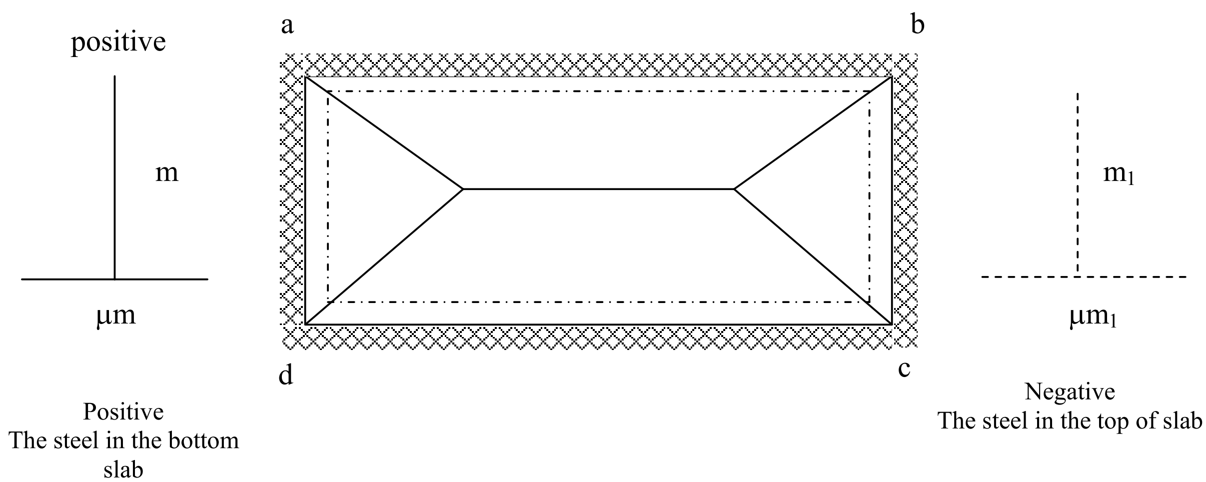


Fig (2.01)

2.2 Solution by Virtual Work[2]:-

Solution by virtual work may be sumarised into five steps.

1) Postulate a yield – line pattern at failure.
2) Choose some convenient points in the slab and give this point virtual displacement δ.
3) The loss of work due to this displacement of the load expressed and calculated from the equation below:

$$\sum W\delta = \text{External work } = \sum\int\int w\delta dxdy$$

W = uniformly distributed load.

δ = Displacement of center of gravity of element

$\int\int dxdy$ = Area of element

4) Calculate t work done in a yield line is the total moment along the yield line multiplied by rotation of the yield line hence the total work done in all they yield lines is given by:

$\sum m\theta$ = Internal work = $\sum m_b L\theta$

m_b = ultimate moment/ unit length along yield line.

L = the length of a yield line.

θ = the rotation of the yield line.

5) The solution for slab is obtained by equating the loss for slab in external work to gain in internal work, thus.

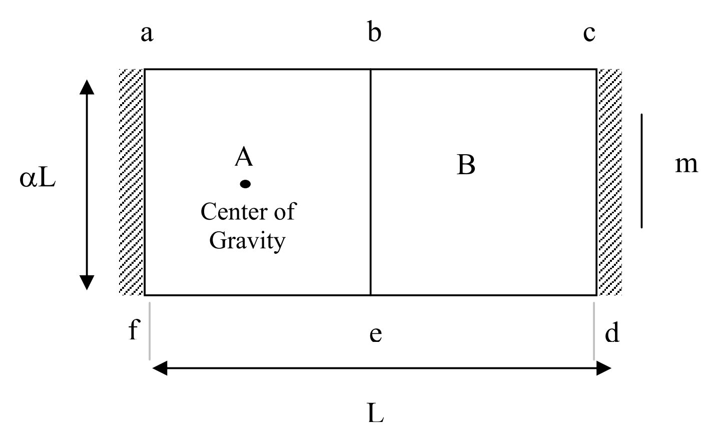
$$\sum\int\int wdxdy = \sum m_b L\theta$$

2.3 Virtual Work Application:-

2.3.1 Solution (1) & (2)

Rectangular slab[1], simply supported along two opposite edges and subjected to a uniformly distributed load of w KN/unit area.

(1) Postulate a yield – line pattern:-



(2) give yield line (be) displacement δ

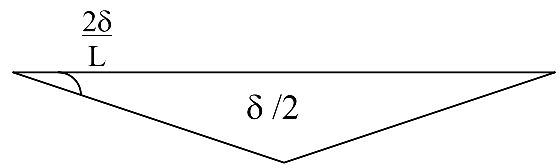


Figure (2.02)

(3) Internal Work:

$\sum (m_b L\theta)$ = Equation (2.02)

Element A = element B

$m_b = m$

$L = \alpha L$

$\theta = \delta / L / 2 = 2\delta/L$

$\therefore \Sigma (M\theta) = 2m\alpha L \times 2\delta/L = 4m\alpha\delta$ (S1. 1)

(4) External Work:-

$\Sigma\iint w\delta\, dx\, dy$: Equation (2.01)

$= \sum W\delta A = 2 \times w \times \frac{1}{2}\delta$

Element A = element B $\Sigma = 2$

$\delta cA = \delta cB = \frac{1}{2}\delta$

$A_A = A_B = \propto L \times \frac{1}{2} L = \frac{1}{2} \propto L^2$

$\sum (w\,\delta) = 2 \times w \times \frac{1}{2}\delta \times \frac{1}{2}\alpha L^2 = \frac{1}{2} w\, \alpha L^2 \delta$ (S1. 2)

(5) $\sum (M\theta) = \sum (W\delta)$

$4m\alpha\delta = \frac{1}{2} w\alpha L^2 \delta$

$M = \frac{1}{8} wL^2$ (S1.3)

Note:-

We observed that the virtual displacement δ dose not enter into the answer, then we can take δ equal unity.

The value of w to be used in the ultimate load equation is the sum of dead load and live load multiplied by their respective load factor.

In (Example) solution (1) that has been obtained, it so happened that the yield line was in the direction of the ultimate moment m the direction of which is usually fixed by practical positioning of the steel. In fact generally the yield line is at an angle to the direction of m, and therefore it is necessary to be able to calculate the ultimate moment in any direction due to the moment m.

Take the yield line shown in figure (2.03) making an angle $\varnothing$ with the direction along which the ultimate resistance moment is m.

The component of the moment due to m in the direction of the yield line is cd $\times$ mcos$\varnothing$, and this is equal to ab $\times$ mb, where mb is ultimate bending moment along the yield line.

I f these values are equated we get.

$m_b \times ab = m \cos\varnothing \times cd$ or

$m_b = m \cos^2\varnothing$

If there is steel in several directions then mb is sum of the values obtained from each of the known moments, thus.

$$m_b = \sum m_i \cos^2 \varnothing_i \qquad (2.04)$$

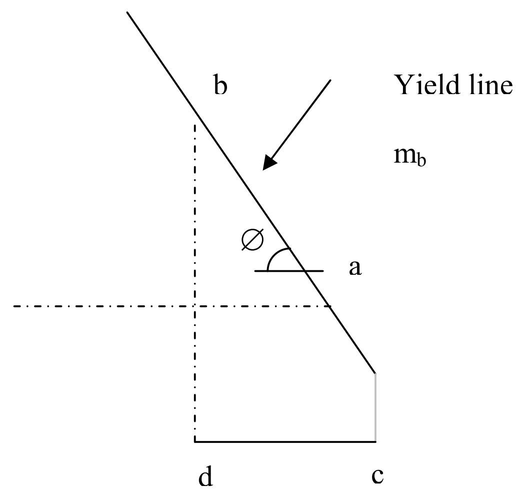
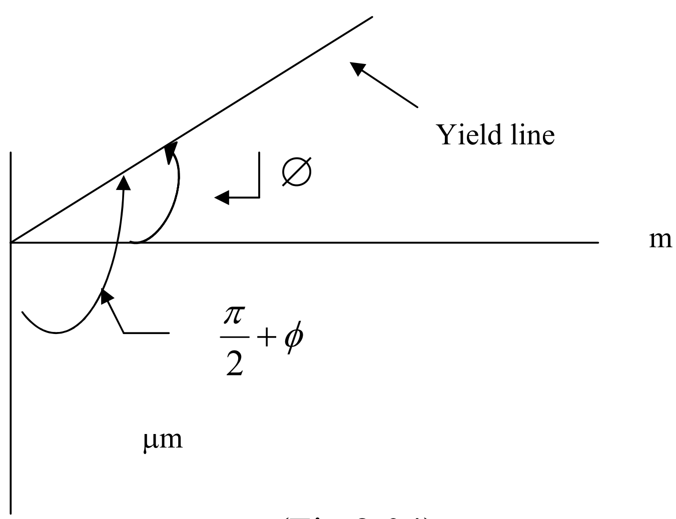


Fig (2.03)

We can use equation (2.04) to find the ultimate moment along the yield line shown in figure (2.04), due to the known ultimate moment/unit length m and μm.



(Fig 2.04)

$m_b = \sum_{i=1}^{n} m_i \cos^2\theta_i$

$m_b = m\cos^2\varnothing + \mu m\cos^2(\pi/2 + \varnothing)$

$m\cos^2\varnothing + \mu m\sin^2\varnothing$

- Special Case: (Istoric reinforcement)

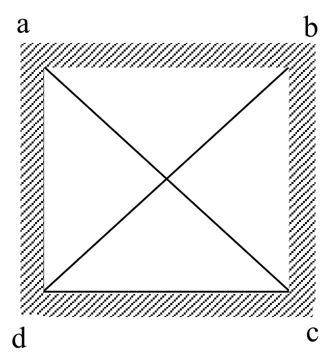
$\varnothing = \pi/2$ and $\mu = 1 \longrightarrow m_b = m$

Thus if a slab is reinforced so that the ultimate moment/ unit length is m in two mutually perpendicular directions. The ultimate moment/ unit length in any directions is also m.[1]

2.3.2 Solution [1]

The simply[2] supported square slab in figure (2.05) is isotropically reinforced with bottom steel such that the yield moment of resistance per unit width of slab is m for bending about any axis.

Determine the required value of m if the slab is to carry a uniformly distributed load of intensity q.



(Fig 2.05)

(1) Postulate a yield line pattern. (Fig 2.05)

(2) Give point e unit displacement.

(3) Internal Work = ($\sum M\theta$):-

$\sum M\theta = \sum m_b L\theta =$

$m_b = m$, $L = L$, $\theta = 1/L/2 = 2/L$

$\sum M\theta = 4 \times mL \times 2/L = 8m$ (S2.1)

(4) External Work: ($\sum w\delta$)

$\sum W\delta = \sum w\delta A$

$\delta = \frac{1}{3}$

$A = \frac{1}{2} \times L \times L/2 = \frac{1}{4} L^2$

$\sum W\delta = 4 \times w \times \frac{1}{3} \times \frac{1}{4} L^2 = \frac{1}{3} wL^2$ (S2.2)

(5) Equated (S2.1) and (S2.2)

$8m = \frac{1}{3} w L^2$

$m = w\frac{L^2}{24}$ (S2.3)

So far, in order to calculate the work done in the yield lines, we have calculated the ultimate moment along the yield line and the true rotation of yield line. Since most of the slabs that are considered are rectangular, the axes of rotation of the slab elements are the slab edges, which are at right angles to each other. In addition the known values of the moments are also in these directions since the steel is placed parallel to edges. Consequently it is generally easier not to calculate the true ultimate moments and the true rotation of the yield lines, but rather to calculate the components of work. This may be expressed as:

$\sum (M\theta) = \sum (M_X \theta_X + M_Y \theta_Y)$

$$= \sum (m_x X\theta_x + m_Y Y\theta_Y) \quad (2.05)$$

Where mx is the ultimate moment/unit length in the direction of the x-axis.

X is the projected length of yield line on the X axis, and θ_x is the rotation of the yield line about the X axis.

The expressions in y is similar.

In order to check the validity of equation (2.05) consider part of a yield line of length L shown as cd in (Fig 2.06a). If the point d has virtual displacement of unity the true rotation of yield line is:

θA + θB and from (Figure 2.06b) the values of these rotations are given by:

$\theta A = \dfrac{1}{L\cot\phi}$ and $\theta B = \dfrac{1}{L\tan\phi}$

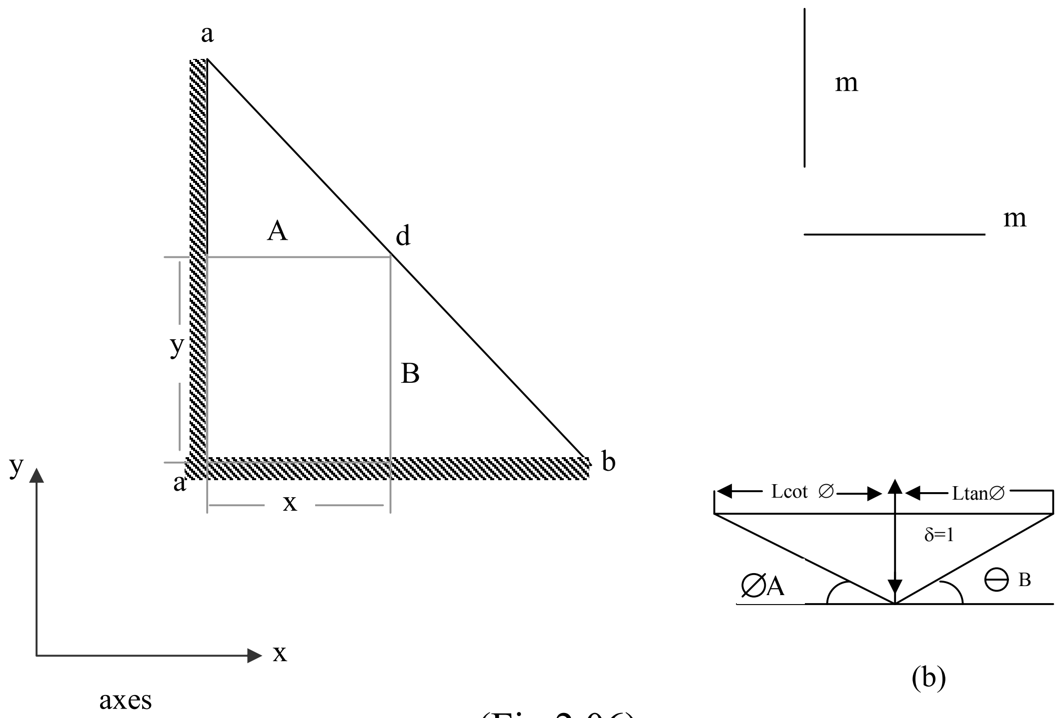
Hence $\sum(M\theta) = ml\left\{\dfrac{1}{L\cot\phi} + \dfrac{1}{L\tan\phi}\right\}$

$= m(\tan\varnothing + \cot\varnothing)$.

Since $m_x = m$, $x = L\cos\varnothing$, $\theta_x = 1/y = 1/L\sin\varnothing$

$m_y = m$, $y = L\sin\varnothing$, $\theta_y = 1/x = 1/L\cos\varnothing$

Then $\sum(m_x X\theta_x + m_y Y\theta_y) = m(\tan\varnothing + \cot\varnothing)$



(Fig 2.06)

Sometimes while the general pattern of failure can be drawn, the exact pattern cannot be determined straight away and there are unknowns (distance) and (angles). In this condition the yield line pattern is defined, and the work equation is obtained in terms of these unknowns, giving an equation of the form.

$M = W\ fi\ (x_1, x_2, \dots x_n)$

Where $x_1, x_2, \dots x_n$ are the unknown we are however only interested in the special pattern that gives the lowest value of w for given m, or alternatively that which for given w requires the largest value of m. This will be when the partial differentials of m which respect to each of the unknowns are zero, thus for the worst pattern we also require,

$$\frac{\partial m}{\partial x_1} = \frac{\partial}{\partial x_1}\left\{ f_1(x_1 x_2 \ldots\ldots\ldots. x_n) \right\} = 0$$

$$\frac{\partial m}{\partial x_2} = \frac{\partial}{\partial x_2}\left\{ f_1(x_1, x_2, \ldots\ldots\ldots x_n) \right\} = 0$$

$$\vdots$$

$$\frac{\partial m}{\partial m_n} = \frac{\partial}{\partial x_n}\left\{ f_1(x_1, x_2, \ldots\ldots\ldots\ldots\ldots x_n) \right\} = 0$$

Thus we see that there are as many equation as there are unknowns and solutions may be obtained.

Chapter Three

3. Yield – Line Analysis By equilibrium

3.1 Introduction:- (1)

Along any yield line there are twisting moments and shear forces as well as the bending moments. The forces and moments, which act on the element on one side of a yield line are equal and opposite to those, which act on the element on the other side of the yield line. These forces and moments are shown in figure (3.01).

The convention that will be adopted is that shear forces will be regarded as positive when acting up wards, while the bending moment mb and the twisting moment mt are positive when they act in the direction shown. It is convertent during this initial stage to represent the moments by vectors. The standard vectors notation being that the moment acts in a clockwise direction when looking along the vectors arrow.

When the yield is positive the bending moment vectors are as shown in figure (3.02a). If the yield negative the direction of the arrows is reversed. Similarly the twisting moment acting along positive yield lines are as shown in figure (3.02b). the value of the bending moment and twisting moment is determined by the a mount and direction of the reinforcement.

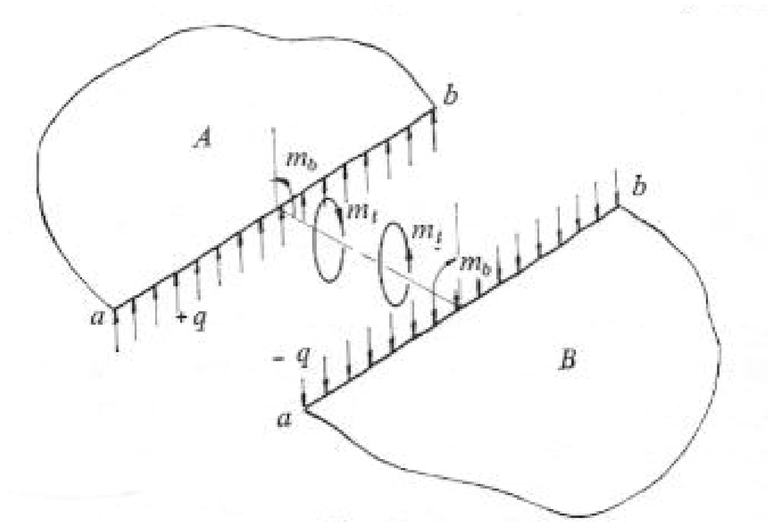


Fig (3.01)

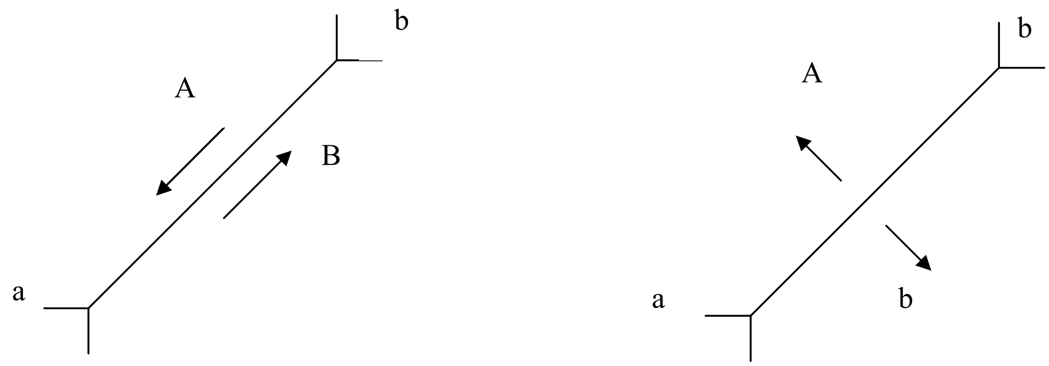


Fig (3.02)

When the method of virtual work is used it is not necessary to know the magnitude or distribution of the reaction or shear forces, which act on the individual elements because the reactions of one element on the adjacent one are equal and opposite and the total work done by these reaction is zero.

When the method of Equilibrium is used it is necessary to know the magnitude of bending and twisting moment and shear force along yield lines, which are the boundaries of elements[1]

3.2 Bending and twisting moments along a yield line:- [1][4]

It is essential to know the magnitude of the bending and twisting moments along yield lines, which are the boundaries of the elements because equilibrium method consider the equilibrium of each of the slab elements.

Let m direction of ultimate moment per unit length intersect yield line (ab) with angle ∅. Which is measured anticlock wise from the direction of m to that of the yield line, this shown in figure (3.03).

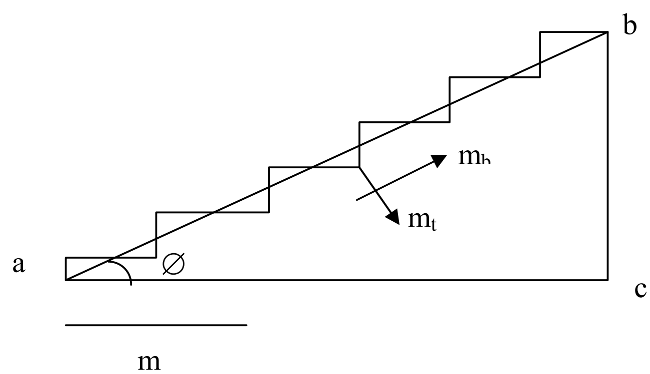


Fig (3.03)

Along the yield line we have the bending moment mb and the twisting moment mt and the resolved components of these must be equal to those values which have just been calculated.

If moments are resolved in direction of moment m.

$$ac \times m = ab \times \cos\phi \times m_b + ab \times \sin\phi \times m_t$$

$m = m_b + m_t \tan\varnothing$ (3.01)

If moments are resolved at right angles to m

$ab \times \sin \varnothing \times m_b$

$= ab \times \cos \varnothing \times m_t$

Then $m_b = m_t \times \cot\varnothing$ (3.02)

Equating (3.01) and (3.02) we get

$m_b = m \cos^2\varnothing$ (3.03)

$mt = m \sin\varnothing \cos \varnothing$ (3.04)

When we have more than one direction the separate effects are merely added together so that:

$$m_b = \sum_{i=1}^{n} m_i \cos^2 \phi_i \quad (3.05)$$

$$m_t = \sum_{i=1}^{n} m_i \sin\phi_i \cos\phi_i \quad (3.06)$$

Let slab reinforced in two directions at right angles such the ultimate moments are m and μm if the angle between the yield line and m is ∅. This is shown in figure (3.04).

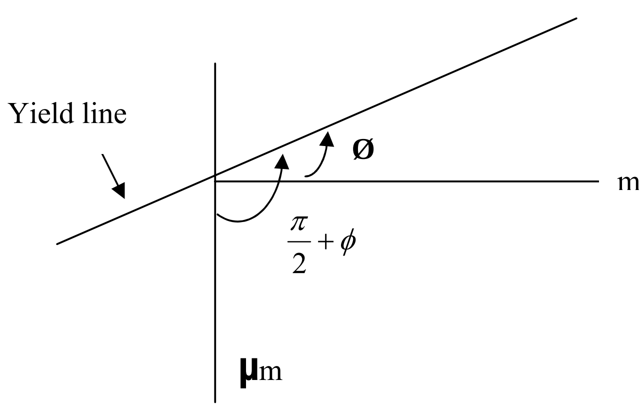


Fig (3.04)

$$m_b = \sum_{i=1}^{n} m_i \cos^2 \phi_i$$

$= m \cos^2\varnothing + \mu m \cos^2 (\pi/2 + \varnothing)$

$= m \cos^2\varnothing + \mu n \sin^2 \varnothing$

And $m_t = \sum_{i=1}^{n} m_i \sin\phi_i \cos\phi_i$

$= m \sin\phi \cos\phi + \mu m \sin(\frac{\pi}{2} + \phi) \cos(\frac{\pi}{2} + \phi)$

$m = \sin\varnothing \cos\varnothing - \mu\, m \sin\varnothing \cos\varnothing$

$= m\,(1 - \mu) \sin \varnothing \cos\varnothing$

For Istropically reinforced slabs: $\mu = 1.0$

$m_b = m$

$m_t = 0$

3.3 Statical equivalents of the shear forces along a yield line:- (1)(4)

Now we replace the actual distribution of forces by two single forces acting at each end of a straight section of yield line. It can be seen that this may be done since any system of co-planar forces can be replaced by two forces acting at chosen positions. These statically equivalent forces are therefore chosen to act at the ends of each straight section of a yield line. Thus for the yield line shown as abc in figure (3.05a)

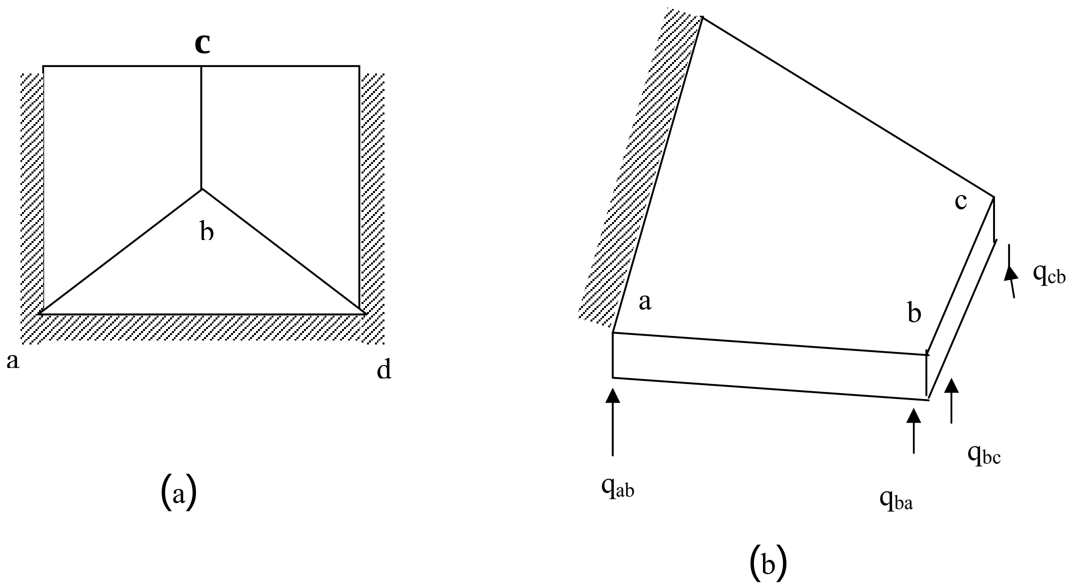


Figure (3.05)

The shear forces along ab are replaced by single forces at a and b while along bc are replaced by single forces at b and c.

Consider three yield lines meeting at a point as shown in figure (3.06). the shear forces acting along yield line ab are replaced by ($+q_{ba}$) at b and ($-q_{ab}$) at a, acting on A, and hence there must be ($-q_{ba}$) at b and ($+q_{ab}$) at a, acting on B. It will be noted that a dot indicates an upward acting force and a cross a downward one. In order to avoid confusion the forces have been drawn slightly away from the point b. The choice of the directions in which the forces act can be quite arbitrary, but the system that has been adopted initially is to have the statically equivalent forces at the ends of a yield line following the direction of the bending moment vector. Thus for yield line ab, the bending moment vector acting on A is to wards corner a so that element A, (q_{ab}) is initially assumed to act downwards and so becomes ($-q_{ab}$) while at b, (q_{ba}) on A acts up wards.

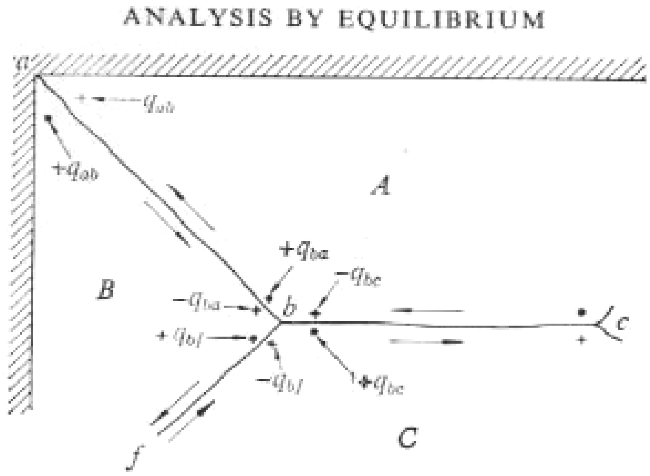


Figure (3.06)

The nodal force acting at b on element A called

QAb equal the sum staticall equivalents.

$QAb = + q_{ba} - q_{bc}$

Also the nodal forces acting at b on element B and C called QB_b and QC_b respectively.

$QB_b = - q_{ba} + q_{bf}$

$QC_b = - q_{bf} + q_{bc}$

If these equations are added together we get

$QA_b + QB_b + QC_b = 0$

Thus we have the important theorem that (at the junction of any number of yield lines irrespective of their sign, the sum of the nodal forces is zero).[1]

We can now proceed to calculate the values of the individual nodal forces, which act at intersection point. In order to calculate the nodal force between any two-yield lines it is first necessary to consider the equilibrium of a small triangular area between these lines.

This small element A' is shown in figure (3.07) where ab and ac are yield lines, but ec is not a yield line. The angle between the yield lines is ∅ where ∅ is measured passing rhrough the element in an anti-clockwise direction. The angle ace is $\delta\varnothing$ and it can be seen that as $\delta\varnothing \rightarrow 0$ the moments which exist along ac also exist along ec.

Let the moments along ec and ac be determined by reinforcing mesh F whose ultimate moments in any arbitrarily chosen direction are mf and μFmF, while the moments along ab are determined by another reinforcing mesh S with it's ultimate moment ms and μs ms in some other arbitrary directions, not necessarily the same as those in mesh f. for convenience the yield lines are at this stage assumed positive. At a, the statical equivalent of the shear forces along ac is (-qac), and that due to the short length ae of yield line ab, qae.

Thus at a due to the shear forces along the short length of yield line ae, and the whole yield line ac we have.

$QAa = q_{ae} - q_{ac}$

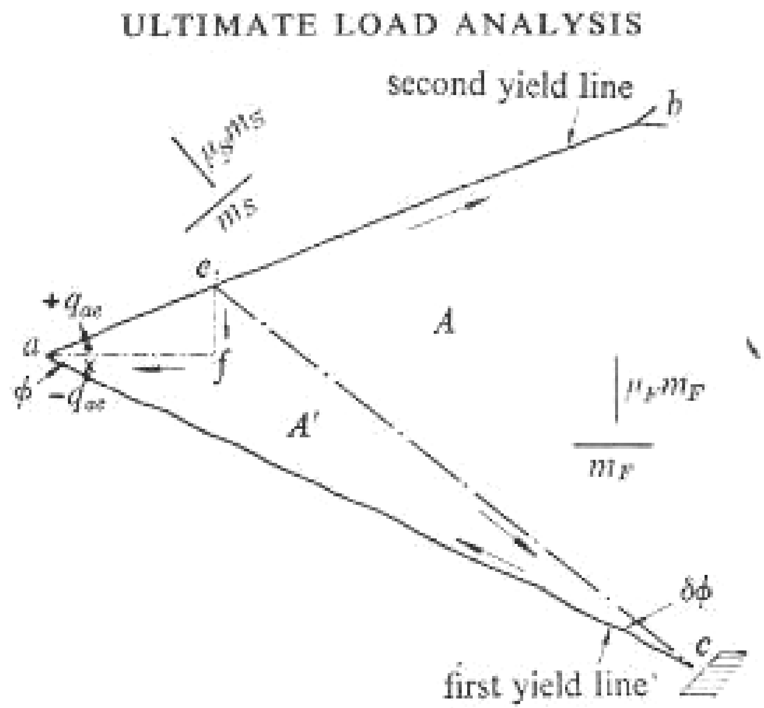


Figure (3.07)

It should be noted that this is not the total nodal force at a, since it only contains forces from part of line ab. In order to calculate QA a, we will be taking moments about the line ec. And consequently the other statically equivalent forces at e and c have not been taken into account, since they have no moment about the axis ec. The paths of the lines ec and ca may be conceived as being stepped first in the direction mF then in the direction of μFmF, is concerned by stepping down to c and back to a is the resultant moment ef × μFmF, where ef is parallel to μFmF and af .

Is parallel to mF. Similarly for mF on ec and ac we have the resultant moment af × mF, the moment act in the sense shown by the vectors along ef and fa.

Thus we have achieved the same resultant moment as if we had taken the path from e to a direct instead of going via c while associating this path, with mesh F.

As before we see that these reultant moments have the same effect as the bending moment mbF and twisting moment mtF acting along ea where mbF and mtf are the bending and twisting moments due to mesh f along the direction ea, similary the moments acting on ae due to µsms and ms are mbs and mts, where mbs and mts are bending and twisting moments along the direction ae in association with mesh S.

Thus the resultant effect of the moments along the boundary ae, ec and ca of element A' is a bending moment (mbs – mbF)s and a twisting moment A(mts – mtFA)s along direction ea the forces to be considered and the resultant moments acting on the element A' are shown in figure (3.08).

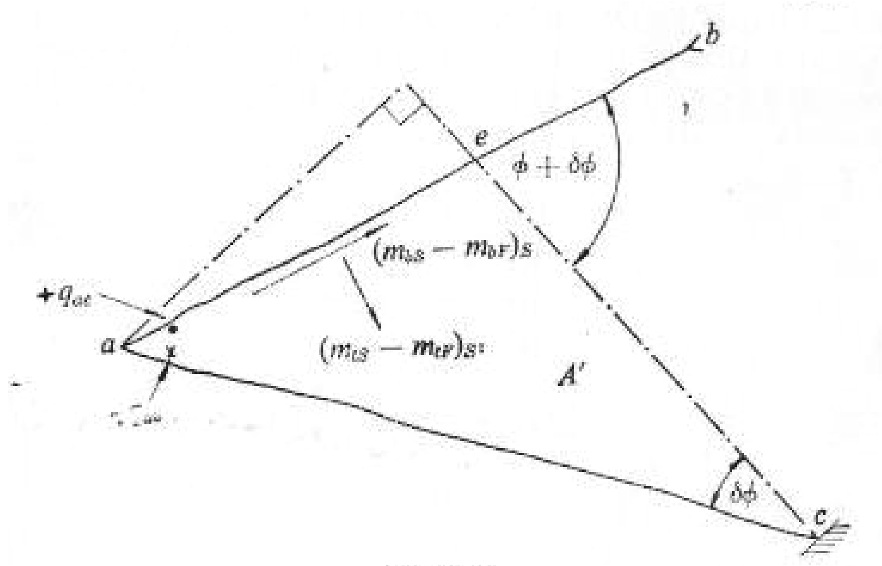


Figure (3.08)

If the moments are taken about axis ec for the equilibrium of triangle A' we get.

$ae\ (q_{ae} - q_{ac})\ \sin(\varnothing + \delta\varnothing)$

$= ae\ (m_{bs}\ m_{bf})s\ \cos(\varnothing + \delta\varnothing) + ae\ (m_{ts} - m_{tF})\ \sin(\varnothing + \delta\varnothing)$

After dividing by ae and when $\partial\phi \longrightarrow 0$ we find that:

$$QA'a = q_{ae} - q_{ac} = (m_{bs} - m_{bF})s\ \cos\varnothing_{Fs} + (m_{ts} - m_{tF})s \qquad (3.08)$$

Where:-

- $(m_{bF})s$ is the bending moment due to the reinforcement under the first yield line along the direction of the second yield line.
- $(m_{bs})s$ is the bending moment due to the reinforcement under the second yield line along the direction of the second yield line.

- $(m_{tF})s$ is the tweisting moment due to the reinforcement under the second yield line along the direction of the second yield line.
- $\varnothing_{Fs}$ is the angle between the first and second yield lines passing through the small element in an anti – clockwise direction.

Now by definition and from figure (3.09)

$QAa = q_{ab} - q_{ac}$ (3.11)

If we substitute for q_{ab} and $- q_{ac}$ into equation (3.11) from equations (3.09) and (3.10) we get:

$QA_a = Q_{12} = (m_{b3} - m_{b1})3 \cot\varnothing 13 - (m_{b3} - m_{b2})3'$
$\cot\varnothing 23 + (m{=}_2 - m_{t1})3$ (3.12)

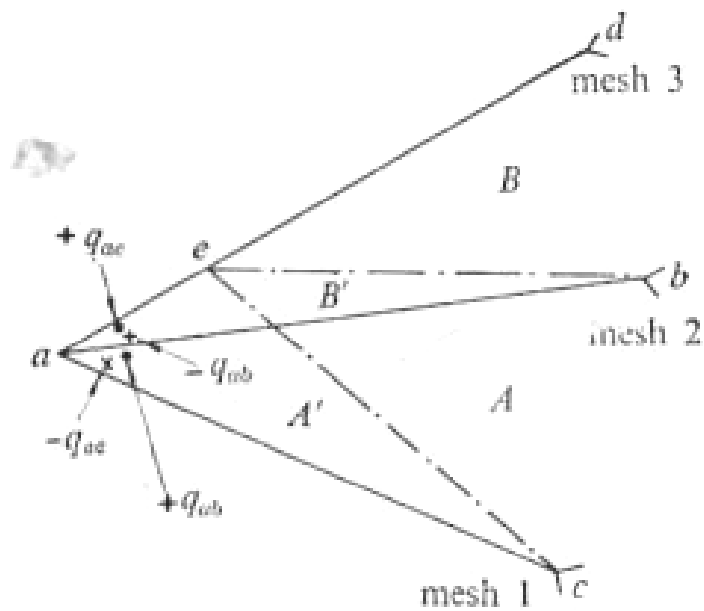


Fig (3.09)

Consider three yield lines meeting at a point and let their ultimate moments be determined by three meshes, 1, 2 and 3. these yield lines are shown in figure (3.09).

We will first consider the equilibrium of the small element A' bounded by ae, ec and ac. The first and second yield lines boundering this element are ac and ad and their ultimate moments are determined by meshes 1 and 3 respectively. Thus for this element the first mesh f is 1 and the second mesh s is 3. So that equation (3.08) becomes:

$$QA'a = q_{ae} - q_{ac} = (m_{b3} - m_{b1})_3 \cot\varnothing_{13} + (m_{t3}\text{-}m_t1)_3 \qquad (3.09)$$

If we now consider the equilibrium of the element B', bounded by ae, eb and ab, then the first and second yield lines for this element are ab and ad and these correspond to meshes 2 and 3.

So that for element B', in equation (3.08) F is replaced by 2 and s by 3 and we get:

$$QB'a = q_{ae} - q_{ab} = (m_{b3} - m_{b2})_3 \cot\varnothing_{23} + (m_{t3} - m_{t2})_3 \qquad (3.10)$$

Now by definition and from figure (3.09):

$$QAa = q_{ab} - q_{ac} \qquad (3.11)$$

If we substitute for q_{ab} and $- q_{ac}$ into equation (3.11) from equation (3.09) and (3.10) we get:

$$QAa = Q_{12} = (m_{b3} - m_{b1})_3 \cot\varnothing_{13} - (m_{b3} - m_{b2})_{3'} \cot\varnothing_{23}$$
$$+ (m_{t2} - m_{t1})_3 \qquad (3.12)$$

3.3.1 Case: (1) (4)

Yield lines all governed by the same mesh (1)(4)

If all the yield lines, which meet, are governed by the same mesh. Shown figure (3.10) such points are marked.

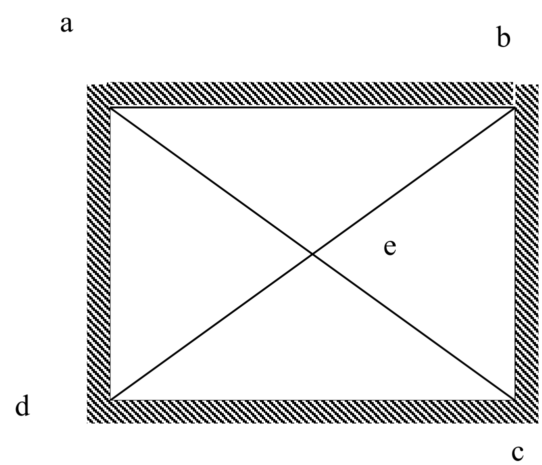


Fig (3.10)

$m_1 = m_2 = m_3$ and $\mu_1 = \mu_2 = \mu_3$

Hence $(m_{b1})_3 = (m_{b2})_3 = (m_{b3})_3$ and $(m_{t1})_3 = (m_{t2})_3 = (m_{t3})_3$

Substituted into equation (3.12)we find that Q12 = 0, similarly by renumbering the yield lines it can be shown that each of nodal forces is zero. This is true whether the yield lines are all positive or all negative and there for leads to the theorem, at the junction of yield lines governed by the same mesh each of the nodel forces is zero.

3..3.2 Case (2):- (1) (4)

Nodal force at the intersection of a yield line with a free edge.

When a yield line intersects a free edge, as in figure (3.11). the nodal forces can be found as follows. If the value of QAb is required, the lines ba, bc and bd are numbered 1,2 and 3. Now lines 2 and 3 are not yield lines but are the free edge and there fore may by classified as yiled lines with zero strength, so that moments in equation (3.12) with their first suffix 2 and 3 are zero and therefore the general equation reduces to:

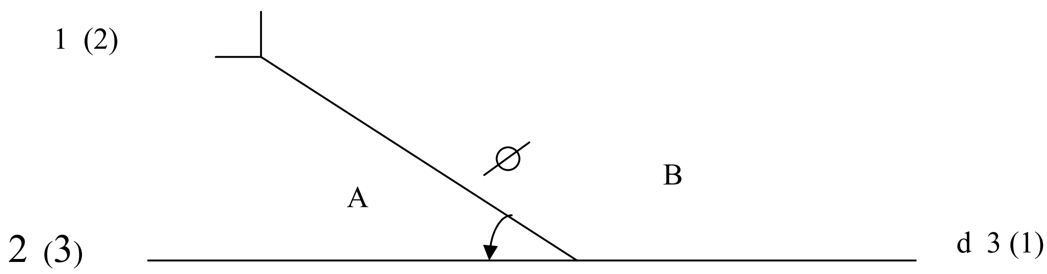


Fig (3.11)

$Q_{12} = Q_{Ab} = -\ \{(m_{b1})_3 \cot \varnothing_{13} + (m_{b1})_3)\}$

Where $\cot \varnothing_{13} = \cot(\pi+\varnothing) = \cot \varnothing$

Since however in these particular circumstances there is only one yield line, the suffix (1) can be dropped, and since direction (3) is the direction of the edge instead of (3) we can use the suffix e thus:

$Q_{12} = Q_{Ab} = -\ (m_{be} \cot\varnothing + m_{te})$ (3.13)

Since the sum of the nodal forces at any point is zero

$Q_{Bb} = -Q_{Ab} = m_{be} \cot\varnothing + m_{te}$

This can easily be checked from the general equation (3.12). In order to find Q_{Bb} the lines bd, ba and bc are numbered (1), (2) and (3) these figures being shown in brackets in figure (3.11).

3.4 Solution by equilibrium:-

The steps which are taken when using the equilibrium:

(1) Postulate a failure mechanism.

(2) Calculate the values of any nodal forces that are required.

(3) Obtain equilibrium equation by taking moments about appropriate axes of rotation are resolving vertical forces for each slab element.

(4) Eliminate the unknowns from the equilibrium equation to obtain a solution.

When considering the equilibrium of an element the element is separated from the remainder of the slab, and the load, nodal forces and moments along the yield lines around this element are in equilibrium. When dealing with the moments in the yield lines there is no longer any need to think of them in terms of bending moments (m_b) and twisting moment mt, it was only necessary to think of them in this manner in order to find the nodal farces. The yield line can be though to be stepped in the direction of the known moment as is shown in figure (3.12) so that the moment due to (m) is de x m or ml cos∅, and the component of this about the axis ab is, ml $\cos^2∅$.

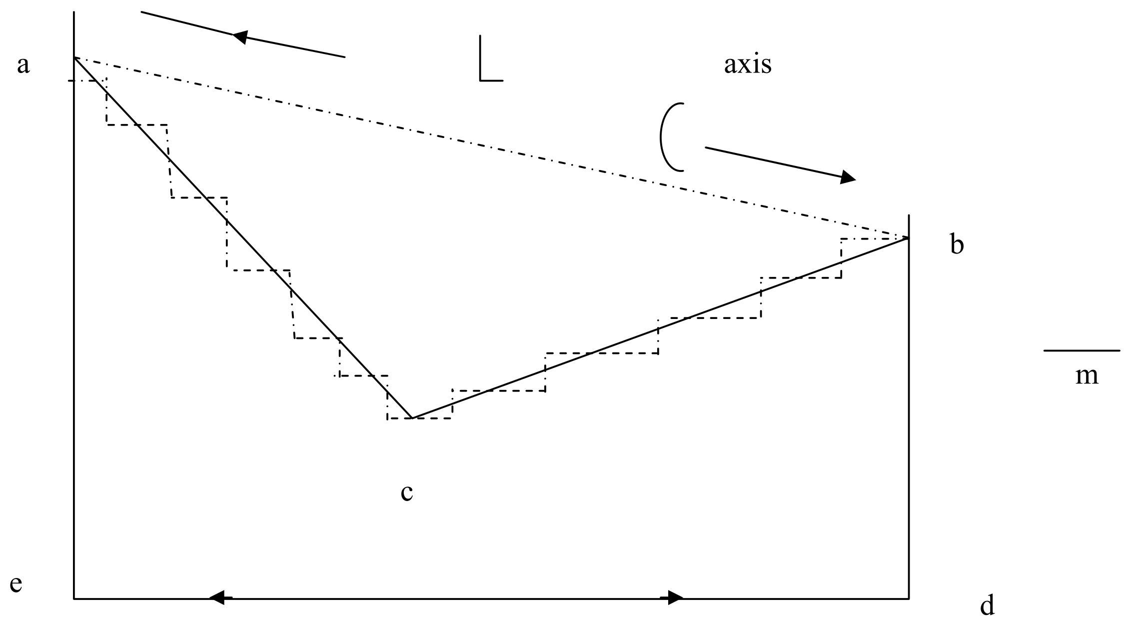


Figure (3.12)

If there are several direction of steel under the yhield line the axis is given by:

$$M = L\sum_{i=1}^{n} mi\cos^2 \phi_i$$

Where $∅_i$ is the angle between the direction of mi the axis about which moments are being taken.[1]

Chapter Four

4.1 Design by yield line theory:-

Design load and moment of resistance in design the problem is to determine ultimate moment of resistance per unit width required for slab with known dimensions boundary conditions, and factored (ultimate) load, the factored load being the required service loads multiplied by the load factors.

For gravity loads according to 1977 ACI code the factored (ultimate load) is

$$U = 1.4D + 1.7L \qquad (4.01)$$

Where D is the service dead load and L the service live load.

Reinforcement is provided for design moment if the ultimate resistance moment per unit width in a particular direction is to be M_u then the design equation for steel in that direction:

$$M_u = \varnothing A_S f_y (d-0.59 A_s f_y/f_c) \qquad (4.02)$$

Where:-

$\varnothing$ = is strength reduction factor taken by the 1977 ACI code as 0.9

A_S= The area of tension steel per unit width.

f_y = The steel yield strength

d = the effective depth to the tension steel

f_c = the compressive cylinder strength of the concrete.

The effect of compression steel on the flexural strength is negligible and may be neglected.

4.2 Reinforcement ratios:-

According to the 1977 ACI code the spacing of bars at critical sections should not exceed twice the slab thickness, and the minimum amount of steel placed in the directions of the spans should not be less than that required for shrinkage and temperature reinforcement. This minimum amount is either 0.002 of the gross concrete area if grade 40 (f_y=276N/mm^2) or grade 50(f_y=345N/mm^2) deformed bars are used, or 0.0018 where grade 60 (f_y=414n/mm^2) deformed bars or welded wire fabric are used or 0.0018*60000/f_y but not less than 0.0014 where reinforcement with f_y > 60000 psi (414N/mm^2) is used.

4.3 Reinforcement Arrangements:-

Yield line theory allows the designer freedom to choose arrangement of reinforcement, which lead to simple detailing. However, it can not be over emphasized that the arrangements of reinforcement chosen should be such that the resulting distribution of ultimate moments of resistance at the various sections through out the slab dose not differ widely from the distribution of moments given by plastic theory. If large difference between the distribution of ultimate resistance moments and the elastic moments do exist. It may mean that cracking at the service load will be excessive because low steel ratios at highly stressed section may lead to high steel stress and hence large crack widths, such regions of high steel stress at service, load may also result in large deflections, hence, it is important that the designer should maintain a feeling for elastic distribution of bending moments and use it to help decide the ratios negative to positive ultimate. Resisting moments to be used in the two directions

(R.Park & W.L.Gamble, Reinforced Concrete slabs, 1980)(3).

4.4 Example: (4.01):-

Design Istropically reinforcement slab has clear span of 16 ft (4.88m) and 24ft (7.32m) as in fig (4.01).

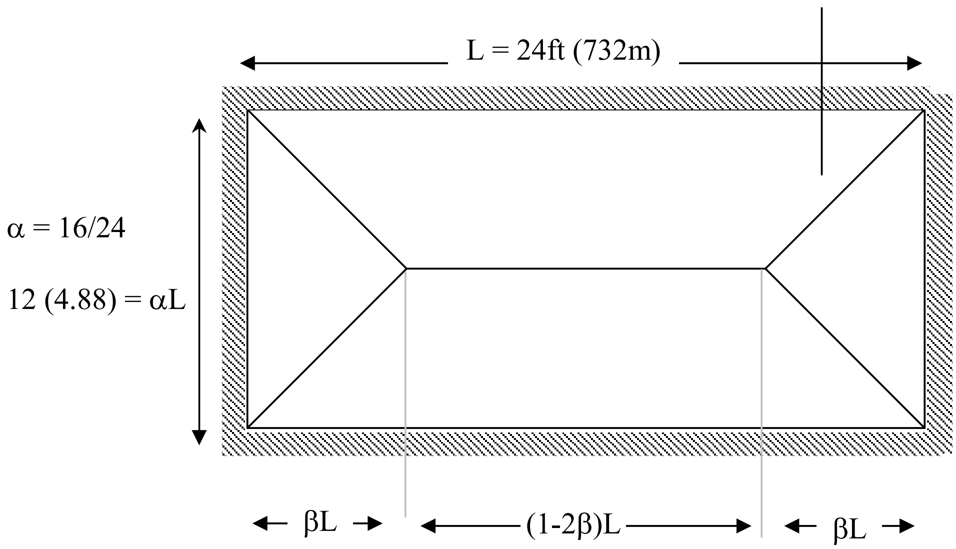


Fig (4.01)

The panel carries a uniformly distributed service live load of 150 psf (7.18 kN/m^2) the panel concrete is of normal weight with cylinder strength of 4000psi (27.6 N/mm^2) and the steel has a yield strength of 60000 psi (413.8N/mm^2) design a suitable panel.

4.1.1 Solution:-

4.4.1.1 Stiffness Requirements:

The minimum slab thickness according to ACI (1318-77) is given by equation (9.7) which for slab aspect ratio of[3]

$\beta = 24/16 = 1.5$

Gives h = Ln/46.4 = 24*12/46.4 = 6.21 in

Say 6.5 in thick slab.

4.4.1.2 Strength Requirement:

Assuming that the unit weight of the concrete is 150 lb/ft^2 the service dead load is

D = 6.5/12 * 150 = 81 psf

The service live load is 150 psf, therefore the factored (ultimate load) according to ACI (318-77)[3] Eq (4.1)

Wu = 1.4 * 81 + 1.7 * 150 = 368psi

The figure 4.01 shows the yield line pattern for the slab the ultimate load of the slab is given by Eq (S_3-4) Chapter 2.

$m = 1/24W\alpha^2 L^2(\sqrt{3} + \mu\alpha^2) - \alpha\sqrt{\mu})^2$

Then w = 24 $m/\alpha^2 L^2(\sqrt{3} + \mu\alpha^2) - \alpha\sqrt{\mu})^2$

The minimum amount of steel allowable is AS = 0.0018 of the gross concrete section

This leads to AS = 0.0018 * 6.5 = 0.0017 in^2/in

Width requiring NO 3 bars on 0.11/0.0117 = 0.94 in, say 9 in centers, giving.

A_S = 0.0122 in^2/in width.

Using NO 3 bars width 3/4 in, in the y direction d = 6.5-1.31 = 5.19 in, placing minimum steel in X direction in the bottom of the slab from Eq (4-2)(3)

Mux = 0.9 * 0.0122 * 60000 (5.19-0.59 * 0.0122 * 60 * 10^3/4000 = 3378 lb .ft/ft width

$Wu = 24m/\alpha^2 L^2(\sqrt{3} + \mu\alpha^2) - \alpha\sqrt{\mu})^2$

α = 16/24 = 0.67

μ = Istoicolly reinforcement = 1

Wu = 24* 3348 / 0.67^2 * 16^2 (3 + 0.67^2) – $0.67)^2$ = 496.15 psf

Which is higher than the required design load of 368psf. There fore minimum steel is suitable reinforcement.

Chapter Five
Theoretical analysis

5.1 Introduction

As indicated clearly in chapter two and chapter three, there are two methods of Yield line analysis of slabs,

(1) Yield – line analysis by virtual work.

(2) Yield – line analysis by equilibrium.

In this chapter analysis is made for four slabs using yield line theory, specifications of slabs are shown below. The full description of the slabs and testing figures are shown in (chapter 6).

Based on the some fundamental assumptions the two methods gives exactly the same results. In either method, a yield line pattern is first assumed so that the collapse mechanism is produced.

For a collapse mechanism, rigid body movements of the slab segments are possible by rotation along the yield lines maintaining deflection compatibility at the yield lines between the slab segments.

5.2 Isotropic and orthotropic slabs:

So far we are dealing with slabs that have had the same amount of bottom reinforcement in each direction at right angles to each other (isotropic slabs). These isotropic slabs are analyzed for the same ultimate positive moments, m, in each direction. In this respect the slight variation in their resistance moments that would result from the differing effective depths is ignored.

In the case of rectangular slabs where there is a marked difference between the two spans it is obviously more economical to span in the short direction and therefore put more reinforcement in the short direction. It is usual therefore to allow a greater moment, m, to develop in the shorter span and a lesser moment μm in the longer span. This then becomes an orthotropic slab, μ is the ratio at the moment capacity in the weaker direction to the moment capacity in the stronger direction, i.e. $\mu < 1.0$. The actual value depends on the designer's choice for the ratio of the two moments or more usually, the ratio of the reinforcement areas in the two direction. At the relatively low levels of moments generally encountered in slabs, the moment capacity is directly proportional to area of reinforcement is valid.

5.3 Specification of slab models:

Four identical two way slabs were fabricated and tested under different end conditions, these were:

Group (1) (A): Fixed supported along opposite long span, simply supported along opposite short span, consist of two types, isotropic and orthotropic reinforcement called (A_1, A_2).

Group (2) (B): Fixed supported along opposite short span, simply supported along opposite long span, consist of two types isotropic and orthotropic reinforcement called (B_1) (B_2).

The slabs had dimensions of 1540 mm × 1175 mm × 60 mm, for the specification details of reinforcement are shown in (5.1- 5.9).

5.4 Application of yield line theory to slabs:

Analysis of the slabs section.

5.4.1 Data for calculation:

(1) Compressive strength of concrete:

$f_{cu} = 48\ N/mm^2$ (average of 12 test (appendix 5)

(2) area of reinforcement (average area 3 specimen) (appendix 3).

$As = 23.706\ mm^2$

(3) Yield strain of concrete $e_c = 0.003$

(4) Modulus of elasticity $Es = 20 \times 10^4\ N/mm^2$

(5) Slab thickness h = 60mm

(6) Concrete cover cv = 10mm

(7) Yield stress of reinforcement $f_y = 386\ N/mm^2$ (from tension test of reinforcement) (Appendix 3).

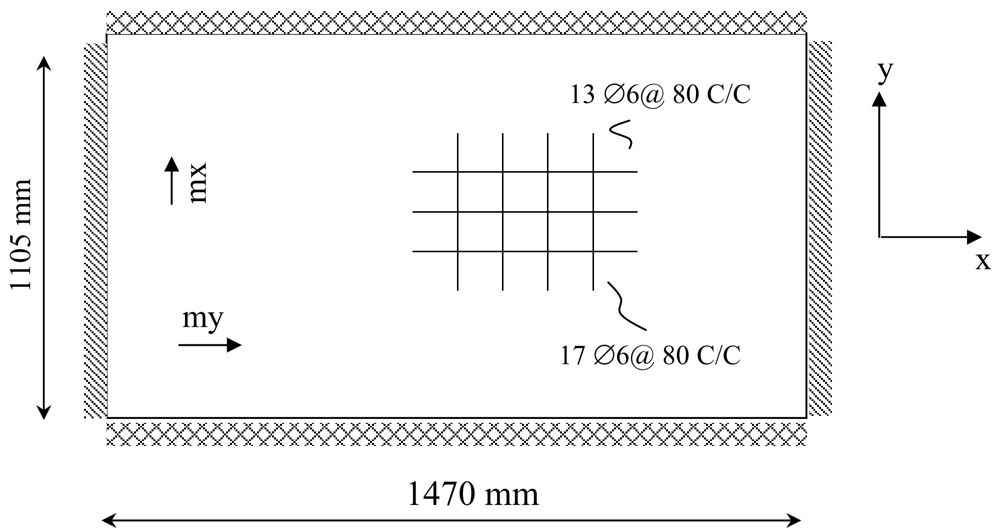
5.4.2 Calculation of the ultimate moments of resistance for slabs.

5.4.2.1 Slab A_1 :

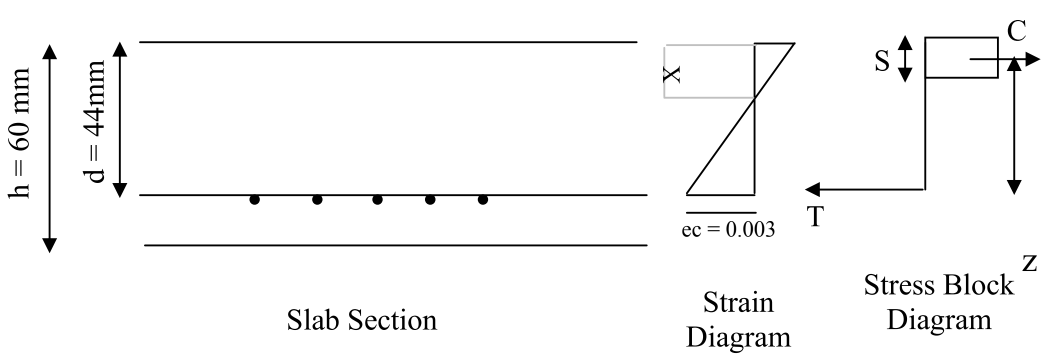
fixed supported slab along opposite long span , simply supported along opposite short span (Isotropic reinforcement).

Resisting moment my:

Referring to fig. (6.1) below and fig. (6.1)



(a)



(b)

Fig (5.1) Slab (A1) Section analysis for **my**

d = 60-6-10 = 44mm (Average effective depth)

As = 17 × 23.705 = 403.00 mm^2

b = 1470 mm

$S = 0.9\ x$

$T = 0.95\ fy \quad As = 0.95 \times 386 \times 403.00$

$C = 0.45 \times fcu \times bs = 0.45 \times 48 \times 1470 \times 0.9\ x$

Equating Forces:

$T = C$

$0.95 \times 386 \times 403 = 0.45 \times 48.0 \times 1470 \times 0.9x$

$X = 5.17$

$S = 0.9\ x = 4.65\ mm$

$Z = d - s/2 = (44 - \frac{4.65}{2}) = 41.675 mm$

$my = 0.95\ fy\ As\ Z$

$= 0.95 \times 386 \times 403 \times 41.675 = 6.1587\ kN.m$

$my = 6.1587/1.470 = 4.18\ kN.m/m$

<u>Resistance moment mx , mx'</u>

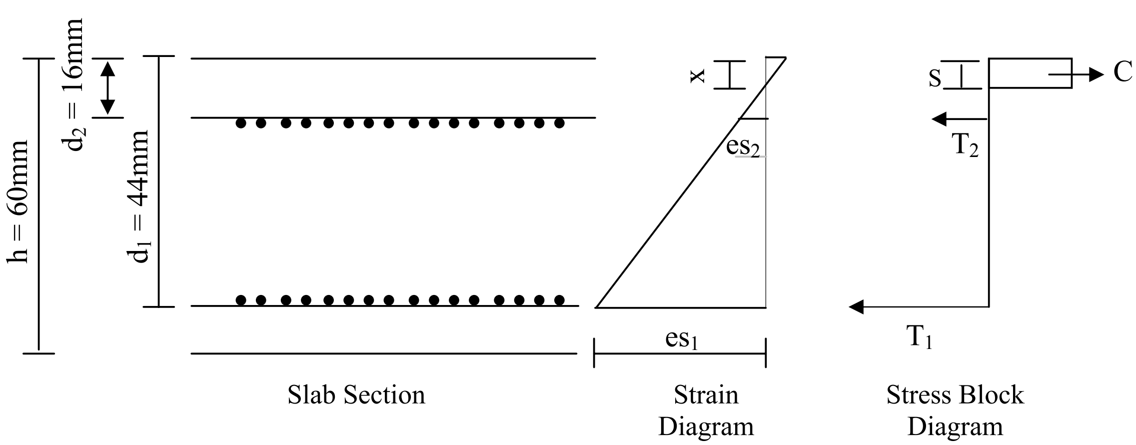


Fig 5.3 Slab A_1 Section analysis

$(mx, mx^{'})$

$A_{sb} = 13 \times 23.706 = 308.178\ mm^2$

$A_{st} = 13 \times 23.706 = 308.178\ mm^2$

B = 1105 mm

$d_1 = 44$ mm

$d_2 = 16$ mm

T1 + T2 = C

T1 = 0.95 As fy

$= 0.95 \times 308.178 \times 386 = 113.01 \times 10^3$

$T_2 = es_2$ Es As

Where:

$es_2 = [(d_2\text{-}x)/x]ec$

$T_2 = [(d_2\text{-}x)/x]$ ec Es As

$[(16\text{-}x)/x] \times 0.003 \times 20 \times 10^4 \times 308.178$

C = 0.45 fcu bs

$= 0.45 \times 48 \times 1105 \times 0.9x = 21.48 \times 10^3$

T1 + T2 = C

$113.01 \times 10^3 + [(16\text{-}x)/x] \times 0.003 \times 20 \times 10^4 \times 308.178 = 21.84x$

$21.48x^2 + 71.89x – 2958.51 =$

X = 10.19 mm

S = 0.9x = 9.17 mm

Taking moment about neutral axis:

$(mx + mx') = 0.45 \times 48 \times 1105 \times 9.17\ (10.19\text{-}9.17/2)$

$+ 0.95 \times 308.178 \times 386\ (44 – 10.19)$

$+ [16-10.19]/10.19] \times 0.003 \times 20 \times 10^4 \times 308.178\ (16-10.19)$

$= 5.6605$ kN.m

$(mx + mx') = \frac{5.66051}{1.105}$ kN.m = 5.12 kN.m/m

5.2.2 Slab A2

Fixed supported along opposite lone span.

Simply supported along opposite short span (orthotropic reinforcement).

<u>Resisting moment my</u>

Referring to Fig (5.3) below and (6.2)

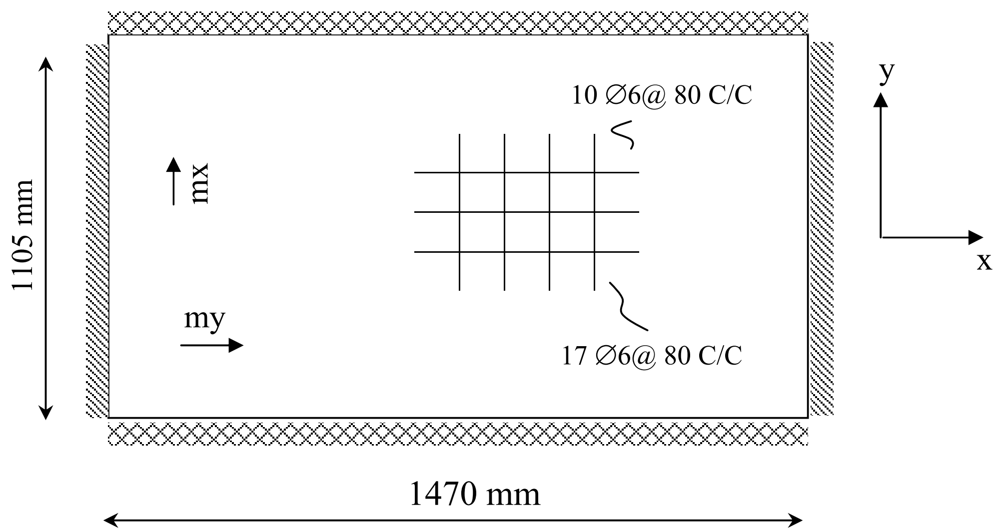


Fig (5.4) slab A_2 section analysis (my)

Resistance moments my:

Resistance moment my is the some as in (A1) in section 6.5.2.1

my = 4.18kN.m

Resistance moment mx, mx':

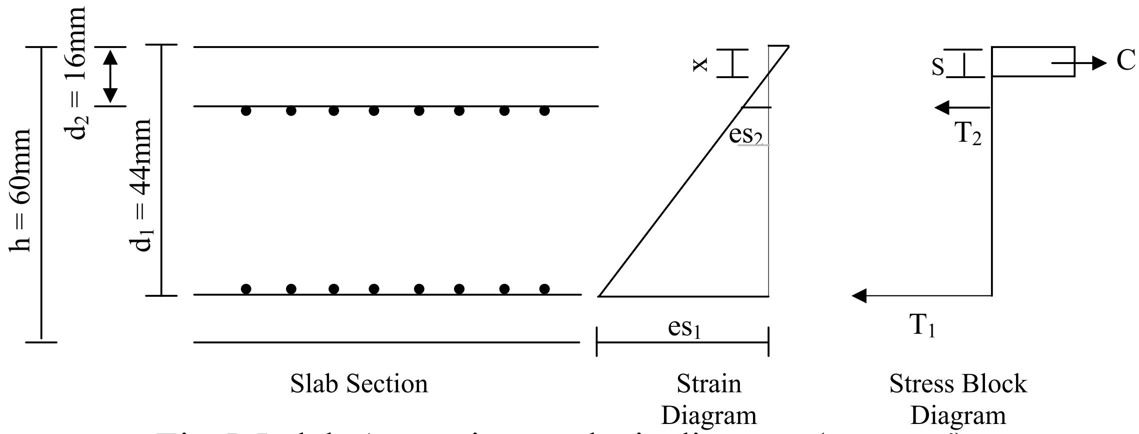


Fig 5.5 slab A_2 section analysis diagram (mx, mx')

Asb = 10 × 23.706 = ×237.06 mm^2

Ast = 10 × 23.706 = 23.706 mm^2

b = 1105m

d_1 = 44 mm

d_2 = 16 mm

$T_1 + T_2 = C$

T_1 = 0.95 Asfy

$= 0.95 \times 237.06 \times 386 = 86.93 \times 10^3$

$T_2 = es_2$ EsAs

$T_2 = [16\text{-}x)/x] \times 0.003 \times 20 \times 10^4 \times 237.06$

C = 0.45 fcu bs

$= 0.45 \times 48 \times 1105 \times 0.95\ x = 21.44x$

$T_1 + T_2 = C$

$86.93 \times 10^3 + (16\text{-}x)/x] \times 0.003 \times 20 \times 10^4 \times 237.06$

$= 21.48 \times 10^3 x$

$= 21.48x^2 + 55.3x - 2275 = 0$

x = 9.083 mm   s= 8.17 mm

Taking moment about neutral axis:

(mx + mx') = 0.45 × 48 × 1105 × 9.17 (9.083-8.17/2) + 0.95 × 237.06 × 386 (44-9.083) + [16-9.083]/9.083] × 0.003 × 20 × 10^4 × 237.061 (16-9.088)

(mx + mx') = 4.88 kN.m

mx + mx' = 4.88/1.105 = 4.4162 kNm/m

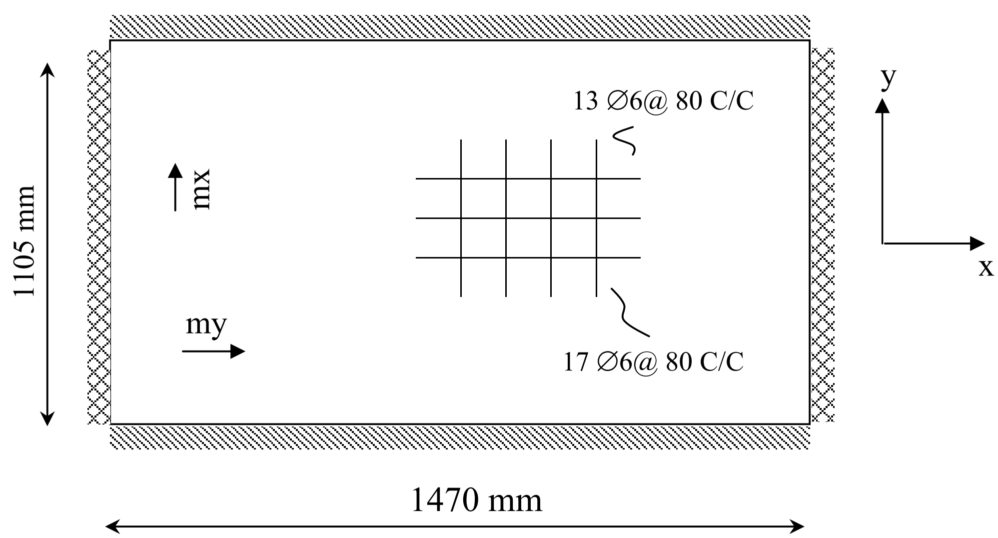
5.5.2.3 Slab B_1

Fixed supported along opposite short span.

Simply supported along opposite long span. (Isotropic reinforcement).

Resisting moment my , my':

Referring to Fig 5.6 below and (6.9)



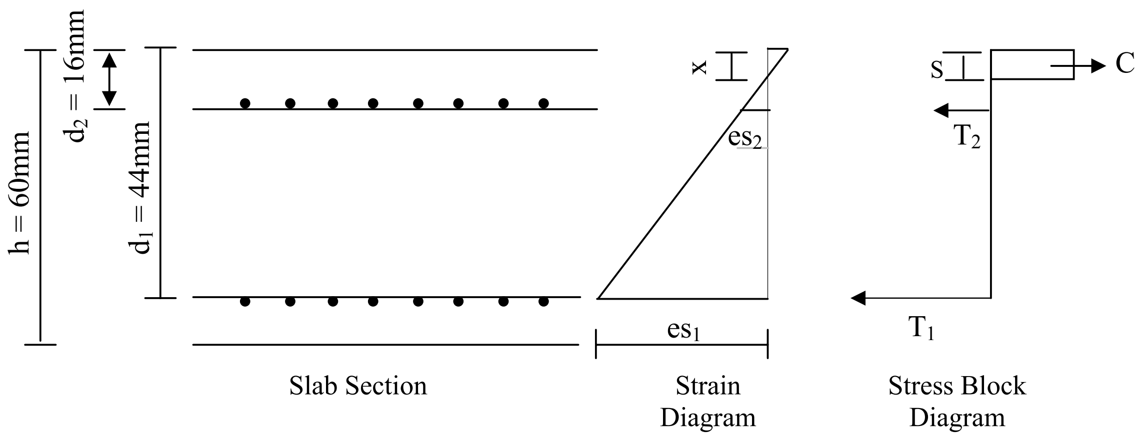


Fig (5.6) slab B1 section analysis (my , my')

Asb = 17 × 23.706 = 403.0 mm2

Asb = 17 × 23.706 = 403.0 mm2

b = 1470 mm

d_1 = 44 mm

d_2 = 16 mm

$T_1 + T_2 = C$

T_1 = 0.95 As fy

$0.95 \times 403 \times 386 = 147.78 \times 10^3$

$T_2 = es_2$ EsAs

Where:

$es_2 = [(d_2\text{-}x)/x]$ ec

$T_2 = [(d_2\text{-}x)/x] \times 0.003 \times 20 \times 10^4 \times 403$

$= [(16\text{-}x)/x] \times 0.003 \times 20 \times 10^4 \times 403$

C = 0.45 fcu bs

$= 0.45 \times 48 \times 1470 \times 0.9x = 28.57 \times 10^3$

$T_1 + T_2 = C$

$147.78 \times 10^3 + [(16-x)/x] \times 0.03 \times 20 \times 10^4 \times 403.0 = 28.57 \times 10^3 x$

$= 28.57 \times 10^3 x^2 + 194.02 \times 10^3 x - 3868.8 \times 10^3$

x = 8.73 mm

s = 0.9x = 7.86 mm

Taking moment about neutral axis:

(my + my') = 0.45 × 48 × 1470 × 7.86 [8.73 – 7.86/2]

+ 0.95 × 403 × 386 (44-8.73)

$+ [16-8.73]/8.73] \times 0.003 \times 20 \times 10^4 \times 403 [16-8.73]$

$+ [16-8.73]/8.73] \times 0.003 \times 20 \times 10^4 \times 403 [16-8.73]$

1.19 + 5.21 + 1.46 = 7.86

(my + my') = 7.86/1.47 = 5.35 kN m/m

<u>Resisting moment mx</u>:

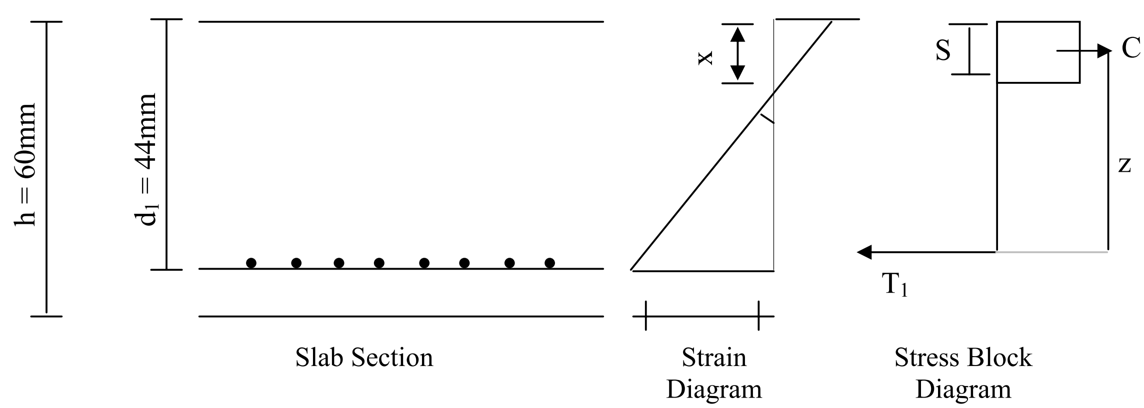


Fig 5.7 slab B_1 section analysis (m_x)

$As = 13 \times 23.706 = 308.178 \text{ mm}^2$

b = 1105 mm

d1 = 44

T = 0.95 × 386 × 308.178

C = 0.45 fcu b s

0.45 × 48 × 1105 × 0.9 x

T = C

0.95 × 386 × 308.178 = 0.4548 × 1105 × 0.9x

x = 5.26 mm

S = 0.9x = 0.9 × 5.26 = 4.734

Z = d – s/2 = 44- (4.734/2) = 41.633 mm

mx = 0.95 f cu As Z

= 0.95 × 386 × 308.178 × 41.633

= 4.738 kN.m

mx = 4.738/1.105 = 4.29 k.N m/m

5.5.2.4 Slab B_2:

Fixed supported along opposite short span simply supported along opposite long sban (orthorropic) Reinforcement

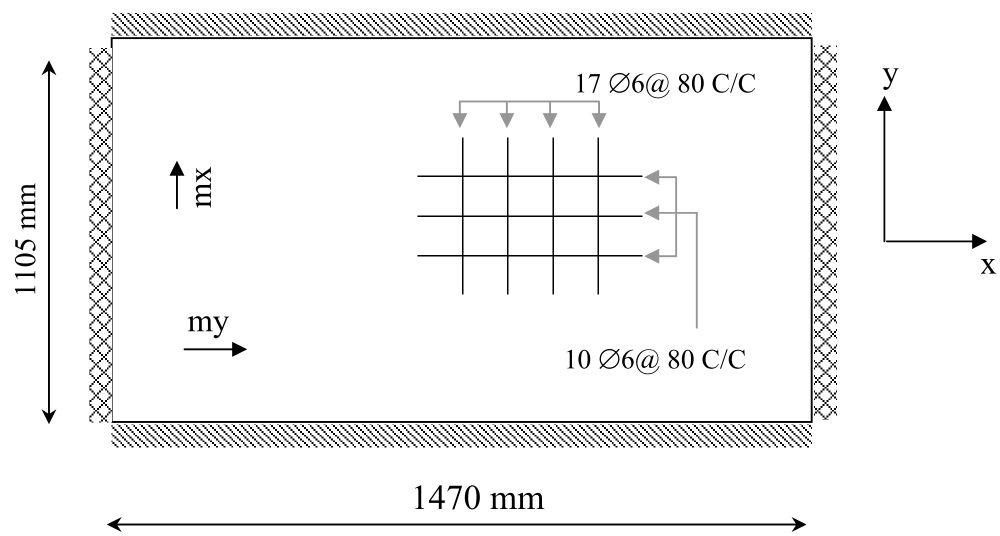


Fig (5.8) slab B_2 section analysis (my, my')

Resistance moments (my, my'):

Resistance moments (my, my') are the same as in (B_1) in section 5.5.2.3

(my+my') = 5.35 kN.m/m

Resistance moment mx

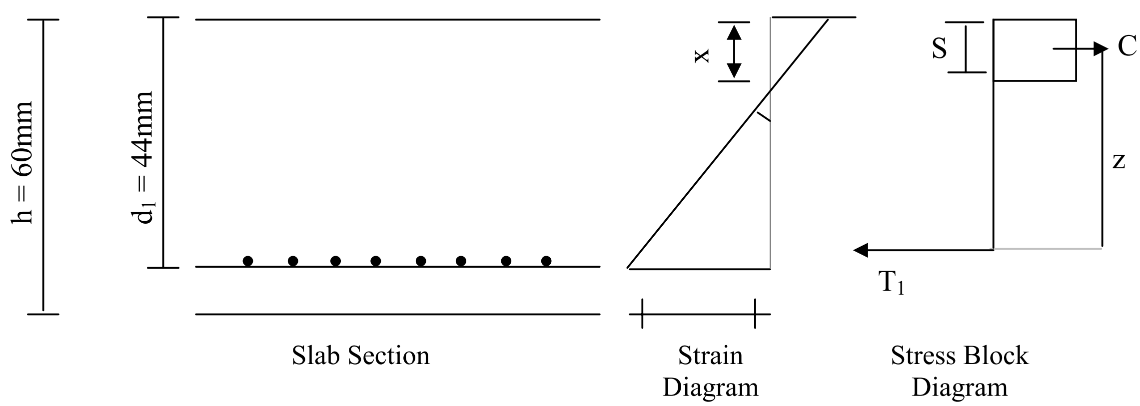


Fig. 5.9 Slab B_2 Section analysis (mx)

As = 10 × 23.706 = 237.06 mm2

b = 1105 mm

d = 44

T = 0.45 × 48 × 1105 × 0.9 x

T = C

0.95 × 386 × 237.06 = 0.45 × 48 × 1105 × 0.9x

X = 4.0467

S = 0.9x = 0.9 × 4.0467 = 3.64 mm

Z = $d - \frac{S}{2}$ = (44 – 3.64/2) = 42.18 mm

mx = 0.95f cu As Z

= 0.95 × 386 × 237.06 × 42.18 =

mx = 3.67 kN.m

mx = 3.67/1.105 = 3.32 kN.m/m

Table (6.1) Ultimate moment of resistance for slabs kN.m/m

Slab Mark	my	mx	(my+my)	(mx+my)
A_1	4.18	-	-	5.12
A_2	4.18	-	-	4.18
B_1	-	4.29	5.35	-
B_2	-	2.32	5.35	-

5.5..3 Yield line analysis:

Analysis by virtual work method

5.5.3.1 Slab A1:

<u>(1) Yield line pattern shown Fig 6.10</u>

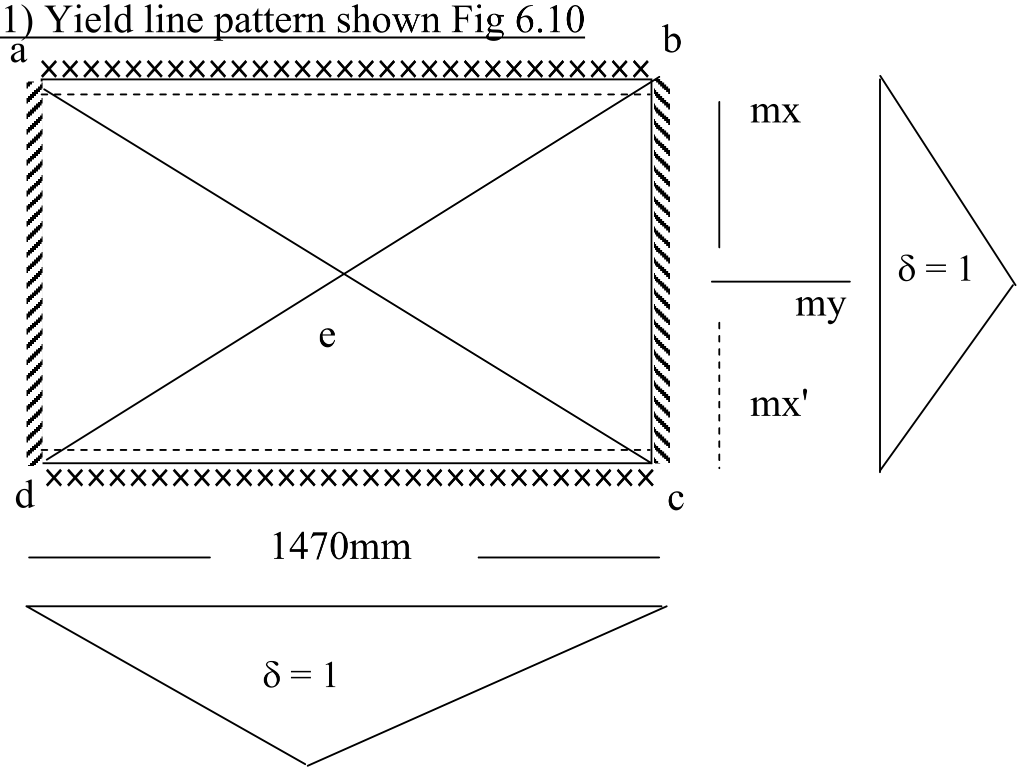


Fig 5.10 Yield line pattern for Slab A1, A2

(2) give point e unit displacement.

(3) Internal work

$$\sum M\theta = 2\left(mx + mx^{1}\right)\times 1470\times\frac{2}{1105} + 2my\times 1105\times\frac{2}{1470}$$

Referring to table 6.1

(mx + mx') = 5.12 my = 4.18 kN.m/m

$$\sum M\theta = 2(5.12)\times 1.105\times\frac{2}{1.47} + 2\times 4.18\times 1.470\times\frac{2}{1.105} = 37.63 KN.m/m \qquad (1)$$

(4) External work

∑wδ = Pu δ = pu (2)

(5) Equating (1) and (2)

Pu = 37.63 kN

5.5.3.2 Slab A2

(1) Yield line pattern shown fig 6.10

(2) give e unit displacement.

(3) Internal work

Referring to table (6.1)

(mx + mx') = 4.46 kNm/m

My = 4.185

$$\sum m\theta = 2(4.46)\times 1.105\times\frac{2}{1.47} + 2\times 4.18\times 1.470\times\frac{2}{1.105} = 32.91$$

(4) External work ∑wδ

∑wδ = Pu (2)

(5) Equating (1) (2) (3)

Pu = 32.91

5.5.3.3 Slab B1

(1) Yield line pattern shown Fig 6.11

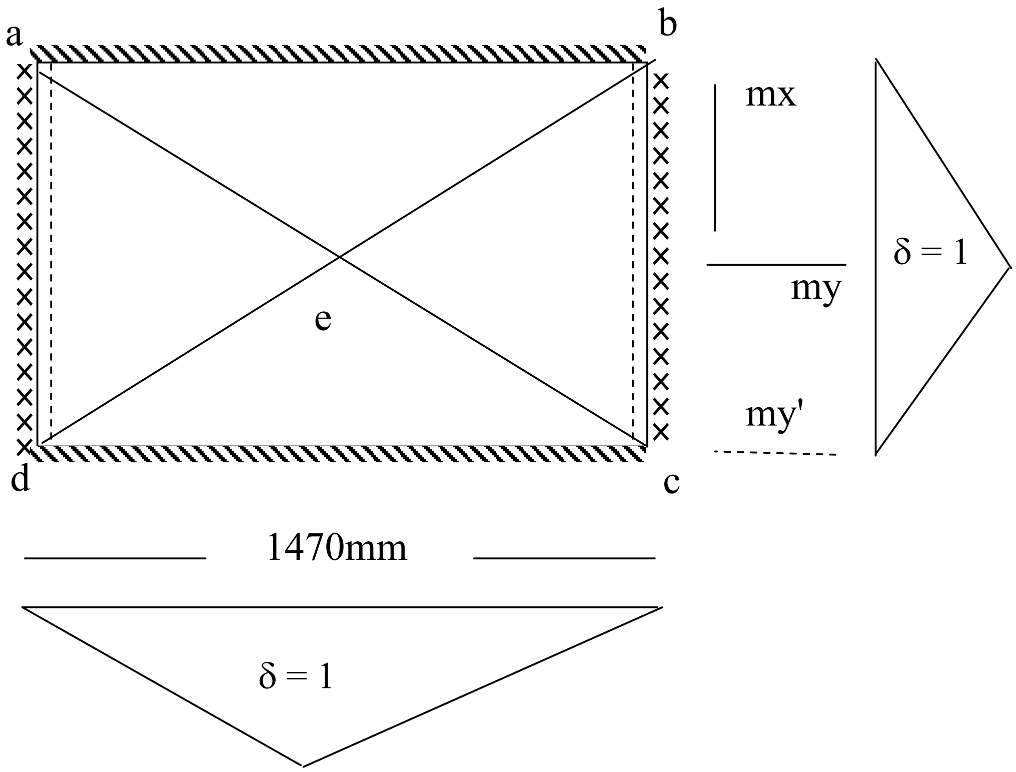


Fig 5.11 Yield line pattern for Slab B1, B2

(2) give e unit displacement.

(3) Internal work $\sum m\theta$

Referring table (5.1)

(my + my') = 5.35 kN.m/m

mx = 4.29 kN. m/m

$$\sum m\theta = 2\left(4.29 \times 1.105 \times \frac{2}{1.47} + 2 \times 5.35 \times 1.470 \times \frac{2}{1.105}\right) = 31.89$$

(4) External work $\sum w\delta$

$\sum w\delta$ = pu (2)

(5) equating (1) and (2)

pu = 31.89 KN

5.5.3.4 Slab B2:

(1) Yield Line pattern shown fig 6.12

(2) give e unit displacement.

(3) Internal work $\sum\varnothing$

Referring to table 6.1

mx = 3.32 kN. m/m

(my+my') = 5.35 kN.m/m

$$\sum M\theta = 2\times 3.32\times 1.105\times \frac{2}{1.47} + 2\times 5.35\times 1.47\times \frac{2}{1.105} = 28.1kNm/m$$

Table (5.2) Theoretical Ultimate Load for Slabs

Slab mark	Ultimate Pu_{theo} (kN)
A1	37.63
A2	32.91
B1	31.89
B2	28.1

Chapter Six

Experimental Work

6.1 Introduction

The theoretical analysis methods and their assumption, which were discussed in previous chapters must be checked by tests before the methods are used in analysis and design.

The purpose of this experimental work is to study the validity of the assumptions made in the yield line theory in slabs under concentrated load subjected at the center. Two categories of slabs are considered. The first category consisted of two slabs fixed supported along opposite long span and simply supported along opposite short span. One of these slab is istropic and the other is orthotropic (referred to as A1, A2). The second category consisted of two slab fixed support along opposite short span and simply supported along long span, one of these slabs is isotropic and the other is orthotropic (Referred to as B1 & B2).

Four tests were conducted in order to verify the theoretical analysis of previous chapter.

6.2 Specification of slabs models:

Two groups of slab are studies, group A and group B.

6.2.1 Group (A) fixed supported along opposite long span:

This group consisted of two slab (A1, A2) each was fixed supported along opposite long spans and simply supported along other spans.

Both slabs were of the same span overall dimensions of 1540 X 1175mm as shown in fig (6.1) and the thickness was 60mm.

Slab (A1) was isotropically reinforced and (A2) was orthotropically reinforcement. Both slab A1, A2 were identically loaded with concentrated load. The reinforcement in these groups consists of one layer along the long span and two layers a long short span. The percentage of steel reinforcement is approximately 0.75% in the istropic reinforcement at bottom, and 0.63% in the orthtropically reinforcement at the bottom. 6mm diameter bars were used spaces at 80mm center to center both way placed parallel to al sides for isotropic reinforcement as shown in fig (6.1) and (6.2) for orthotropic slab reinforcement 6mm spaced at 100mm and 80mm placed parallel to long and short sides respectively as shown in Fig (6.3) and Fig (6.4). The cover of reinforcement is 27.5mm for short span and 50mm for long span, and 10mm for the lower surface of concrete.

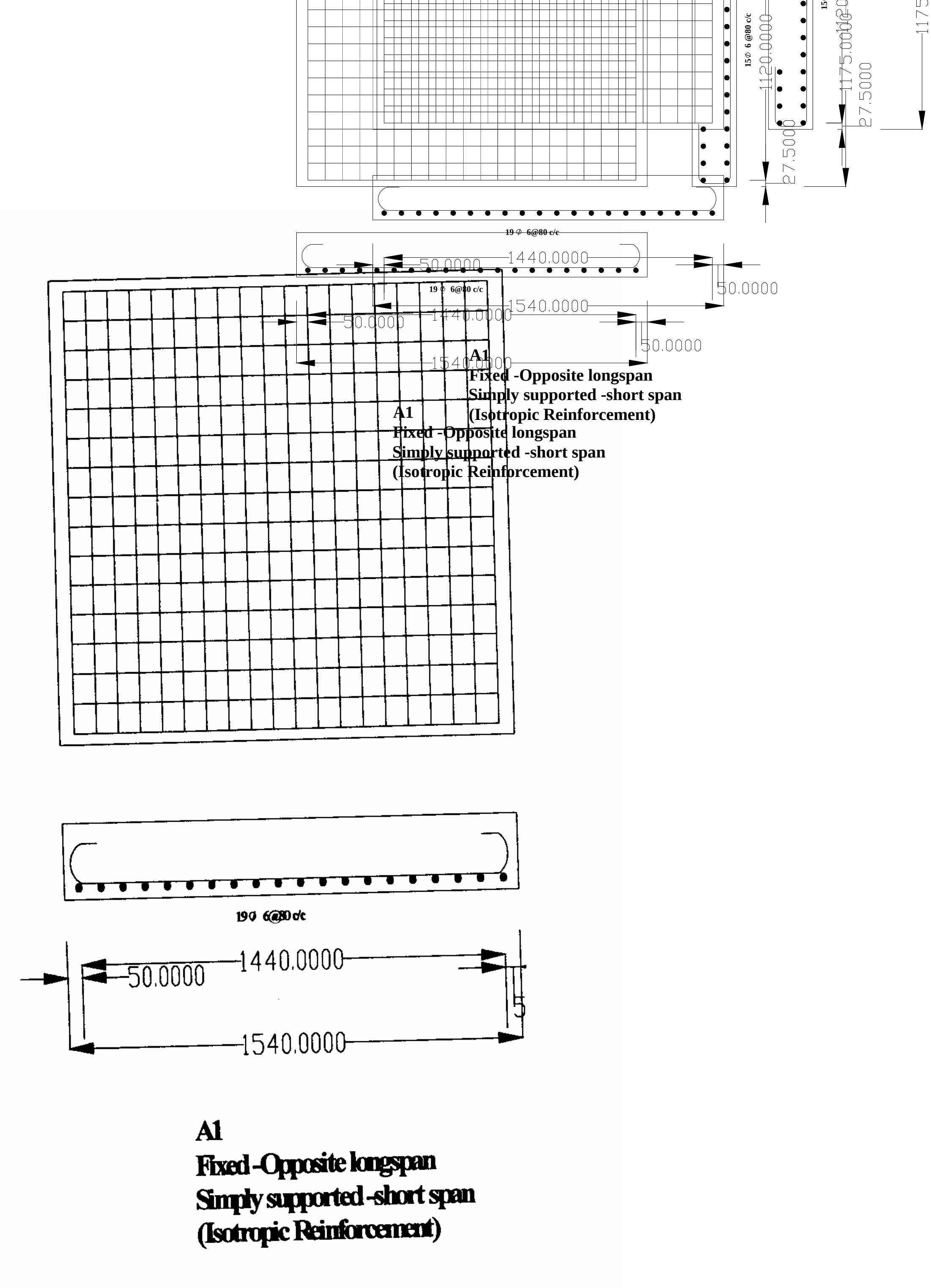
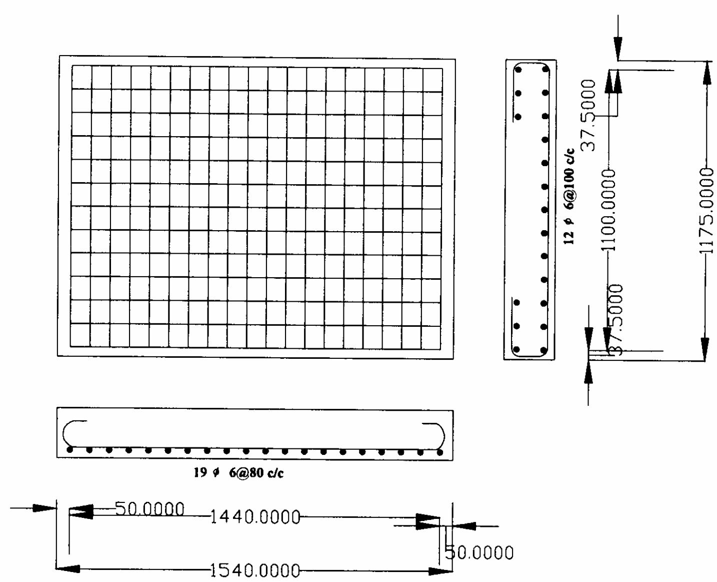

19Φ 6@80 c/c
50.0000
1440.0000
1540.0000
A1
Fixed -Opposite longspan
Simply supported -short span
(Isotropic Reinforcement)

Fig (6.1)

View of the Arrangement of the Reinforcement- for Slab (A1)

(Isotropically Reinforced)

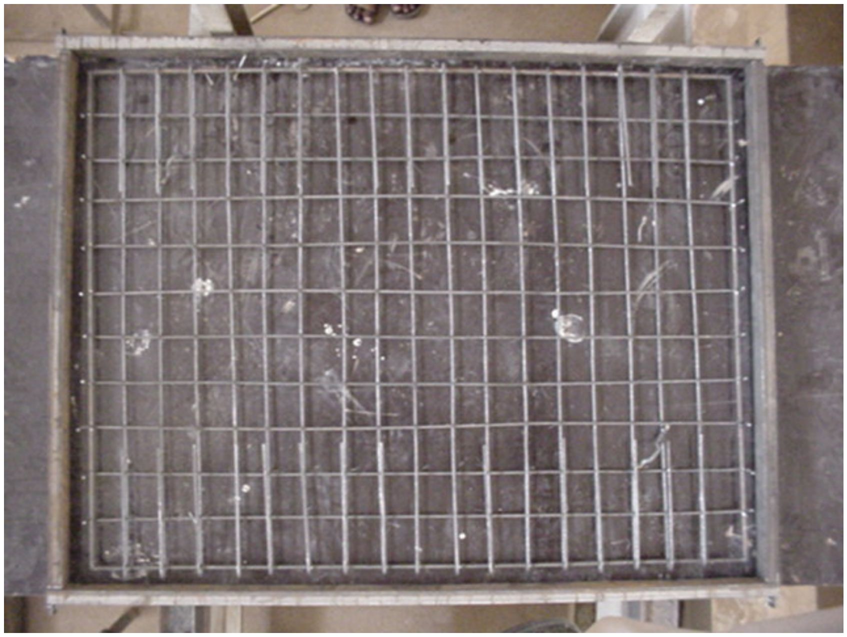
Fig (6.2)



A2
Fixed -Opposite long span
Simply supported -short span
(Orthotropic Reinforcement)

(b)

Fig (6.3)



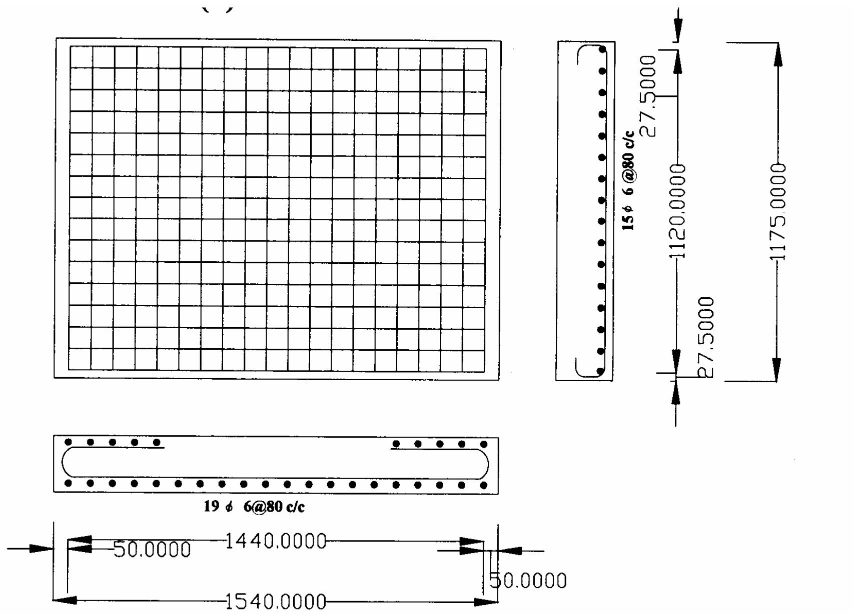
View of the Arrangement of the Reinforcement for Slab (A2) (Orthographically Reinforced)

Fig (6.4)

6.2.2 Group (B) Fixed Supported Along Opposite Short Span:

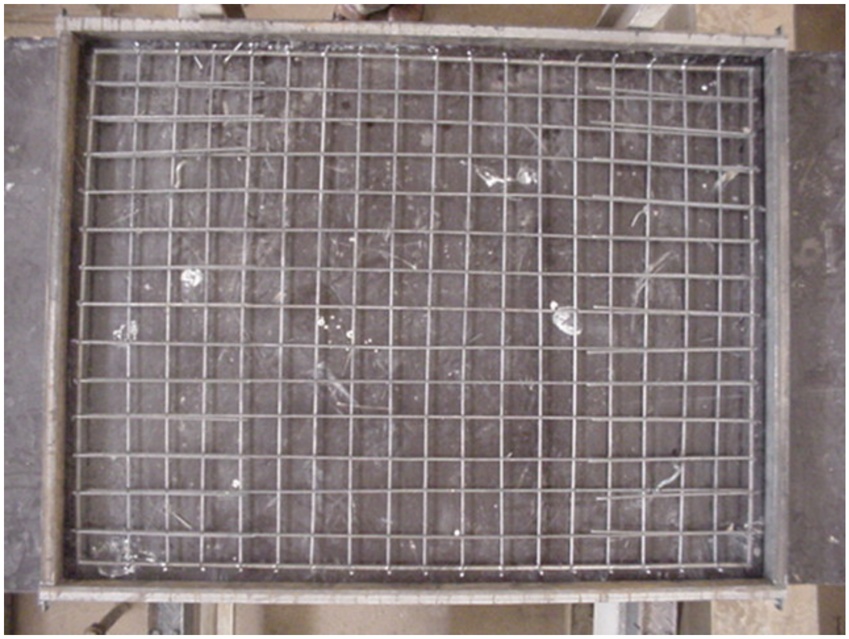
This group consisted of two slabs B1 and B2 each was fixed supported along opposite short span and simply along other span. Its overall dimension is 1540mm × 1175mm and slab thickness is 60mm. The reinforcement is one layer along short span and two layers along other span, top and bottom reinforcement the percentage of reinforcement used in top & bottom is 0.73% for isotropic reinforcement and 0.62 for orthotropic reinforcement, 6mm diameter bar were also used and spaced the same as in group A, as descried in section 7.2.1.

The arrangement of the reinforcement is shown in Fig (6.5), Fig (6.6), Fig (6.7) and Fig (6.8).



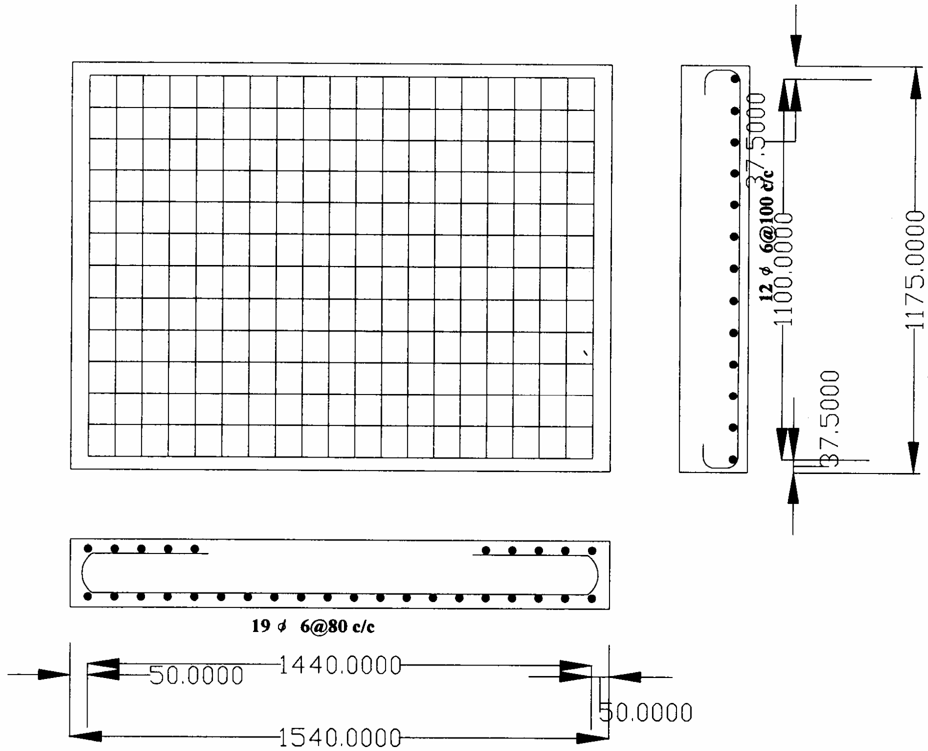
B1
Fixed -Opposite short span
Simply supported -long span
(Isotropic Reinforcement)

Fig (6.5)



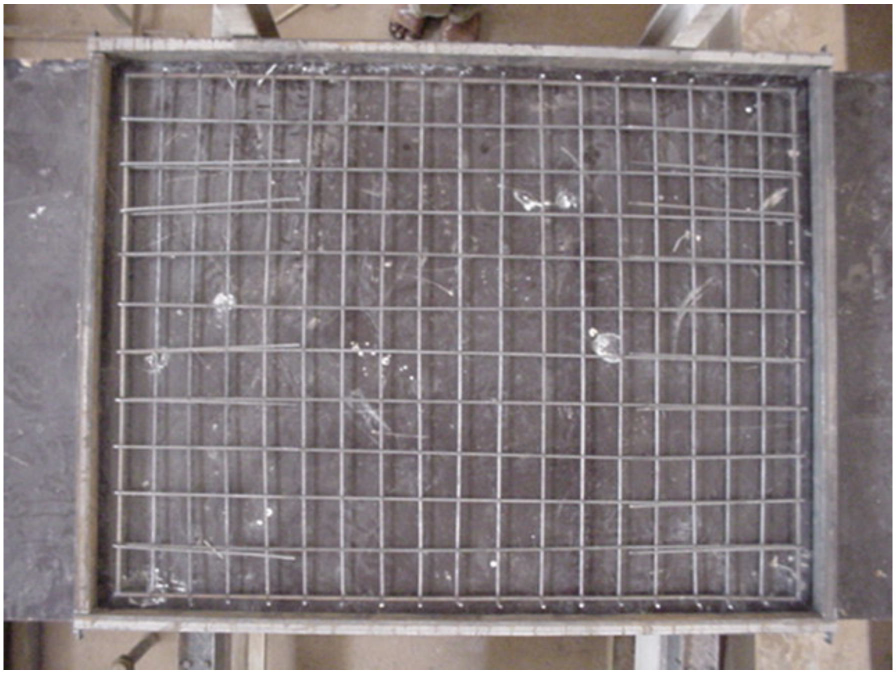
View of the Arrangement of the Reinforcement for Slab (B1) (Isotropically Reinforced)

Fig (6.6)



B2
Fixed – Opposite short spans
Simply supported – long spans
(Orthotropic Reinforcement)

Fig. (6.7)



View of the Arrangement of the Reinforcement for Slab (B2)
(Orthotropically Reinforcement)
Fig (6.8)

6.3 Manufacturing Of Test Models:

The materials used for concrete were:

6.3.1 Cement:

Ordinary Portland cement (marine) complied with standard specification; the consistency of the cement is 28% and the initial and final setting time was found 2^{h}: 52^{m}, 3^{h}: 27m, respectively. Prism compression test of the cement mortar is also carried out and the average crushing strength from three specimens is 18.8N/mm^2 for 2 days and 51.7 N/mm^2 for 28 days (Appendix 1).

6.3.2 Sand:

Sand classification used is Zone 2 carried out from sieve analysis test results (Appendix 2.01).

6.3.3 Coarse Aggregate:

The type of course aggregate used is crushed stones and from sieve analysis test was found that is well uniform (Appendix 2.02). due to small dimensions and thickness of slabs the crushed stone used were of Maximum Size 10mm.

6.3.4 Mix Design:

The mix design was controlled to achieve 30 N/mm^2 at 28 days maintain reasonable medium workability 30-60mm and avoidance of excessive bleeding. Trial mix was done to maintain that mentioned above. Slump test was done in trial mix and the result was 60mm. Also the result for crushing strength list for three specimens, the average value was 29.1 N/mm^2 for 7 days and 43.3 N/mm^2 for 48 days (appendix 5). From these results we conclude that the mix design is acceptable. The proportions of the mix were 1:1.72:2.21 by weight with a water-cement ratio of 0.53.

The concrete was mixed in two batches for each model by using a mechanical mixer. The ingredients for each batch were: 31Kg cement 16 Kg water, 69 Kg gravel and 54 Kg sand.

A mechanical mixer capacity is 250 kg was used for mixing the cement, sand and gravel for about three minutes, dry mixing was done first, while the mixing was going on for about two minutes until suitable consistency of the mix was obtained.

The consistency of the ix was tested by use of the ordinary. Slump test in truncated cone about 300mm high, 100mm top diameters and 150 mm bottom diameter.

6.3.5 Work Form:

A plywood form at the bottom used as mould for all the models and steel frame rectangle Hollow section (6X3mm) used as from to the sides of mould. The steel reinforcement was ordinary plain mild steel bars of 6mm diameter fixed together using wires.

The concrete was placed in the mould within a few minutes from the time of final mixing, manual compaction was used to compact the concrete in the mould. The surface was finally finished by using steel trowels. After 24 hours after placing the concrete, the sides of the moulds were stripped off and the control specimens were also removed from their moulds.

The model was covered together with the control specimens to present evaporation of water. The mould and control specimens were cured by spraying water every day to date of testing.

6.4 Control Test Data:

6.4.1 Preparation of the control specimens:

Six standard steel cubes (10 X 10 X 10mm) were casted with each model to ensure the quality of the concrete. The curing of these control specimens was done to comply with the same conditions applied to the test model and tested on the same day with the model.

Compression test of the standard cube was performed by testing machine. Three cubes were tested to determine the crushing strength of concrete, the test results of the control specimen of concrete are given in Table (6.1) and (Appendix 5).

Table 6.1 Result of Compressive Strength Test 28 days

Slab type	Load (kN)	Compressive Strength for Slabs (N/mm2)
A_1	497	49.7
A_2	482	48.2
B_2	498	49.8
B_2	443	44.3
Average		48

6.4.2 Tension Test of Steel Reinforcement:

Tension test was performed on three specimens of ordinary mild steel bars of 6mm diameter and length 660mm to determine its yield stress, ultimate strength moudulus of elasticity and its deformation (percent of elongation). The results are given in table (6.2) and the stress strain curve in shown in Fig (6.9).

Table 6.2 Results of the Tension Test of Steel reinforcement

Sample	**Yield Strength N/mm^2**	**Ultimate Stress N/mm^2**	**Elongation %**
1	393	444	26.1
2	400	452	25.9
3	364	413	27
Average Value	386	436	26.33

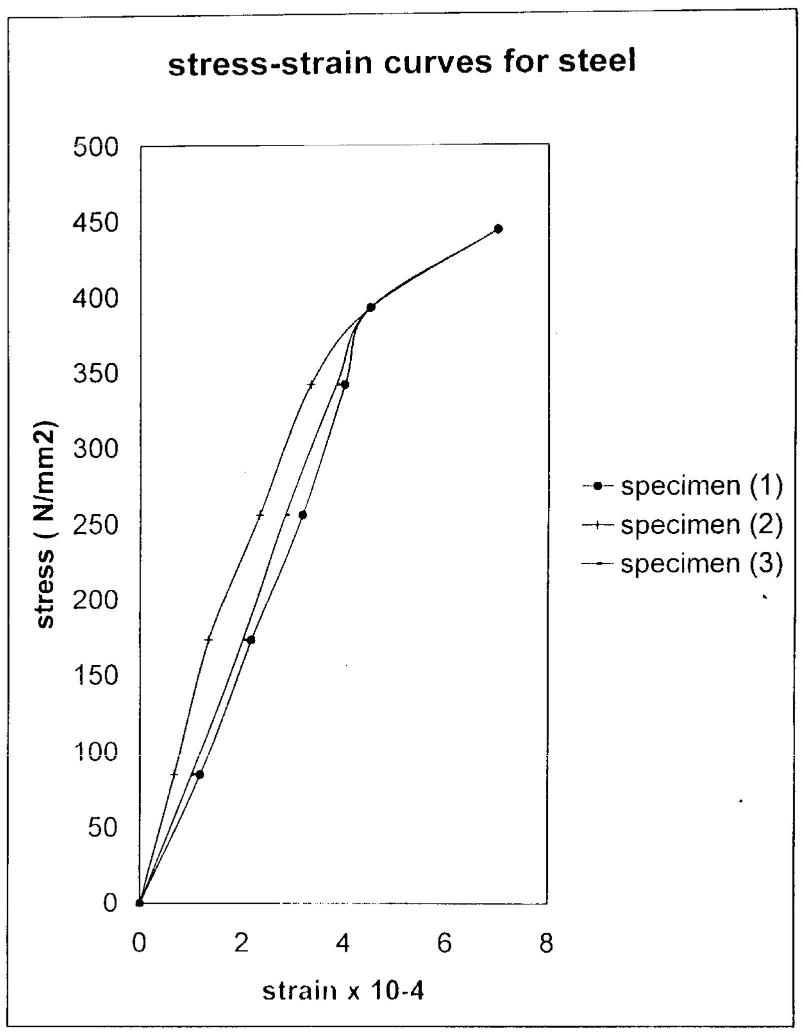


Fig (6.9)

6.5 Experimental Setup :

6.5.1 Testing Frame:

As shown in Fig. (6.10-6.13) the testing frame consisted of four main channels acting as stanchions and connected at top and bottom with four 10" deep channel sections forming a rectangular frame around the stanchions at a height of 80cm above floor level. The top framing channels could be moved freely up or down the stanchions and fixed by means of 3/4" (20mm) bolts in any position to suit the height of the specimen. 16mm diameter bar was welded on top of each to provide a line support, for the slabs.

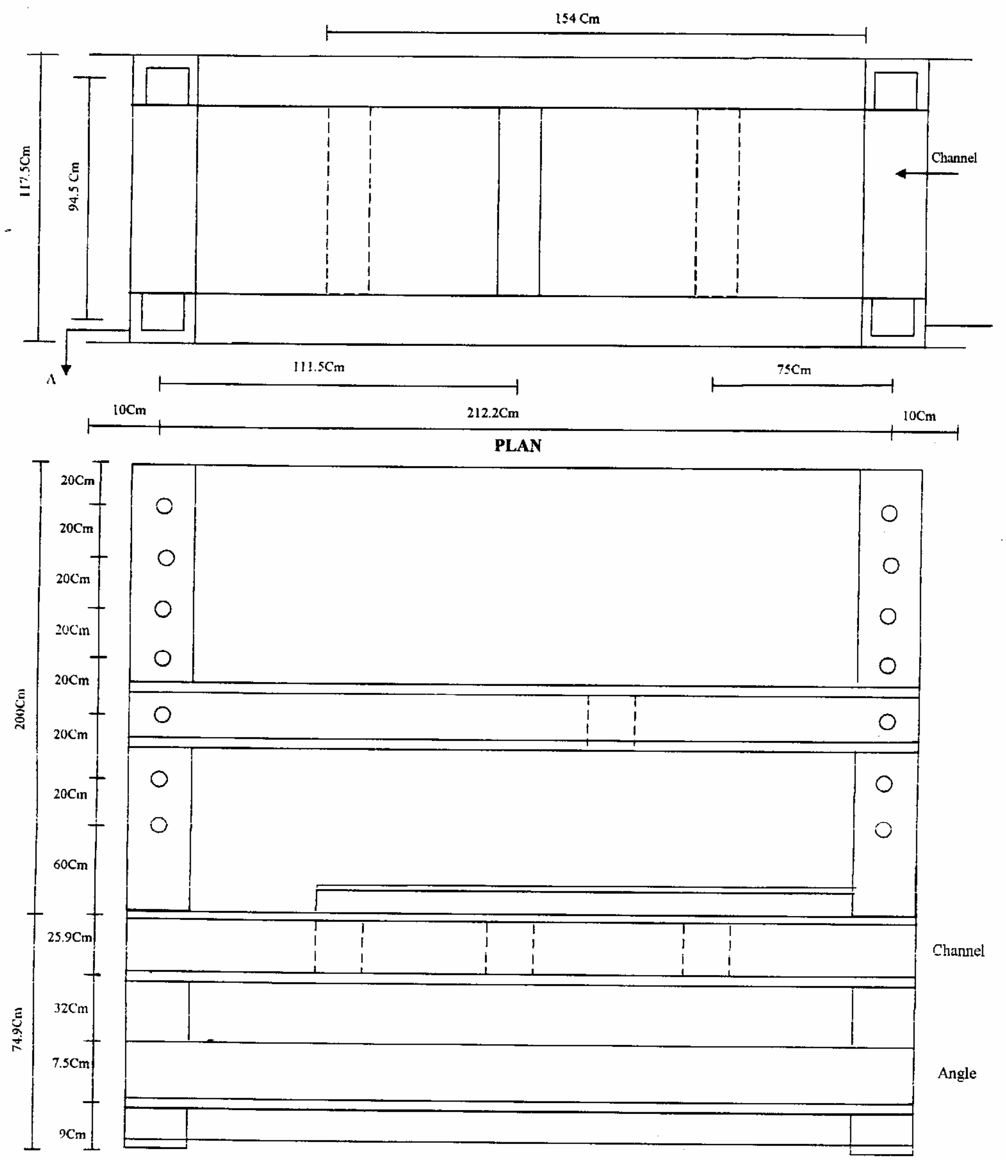


Fig (6.10) Dimension Testing Frame

Fig 6.11 View of Testing Frame

Fig 6.12 View of Testing Frame

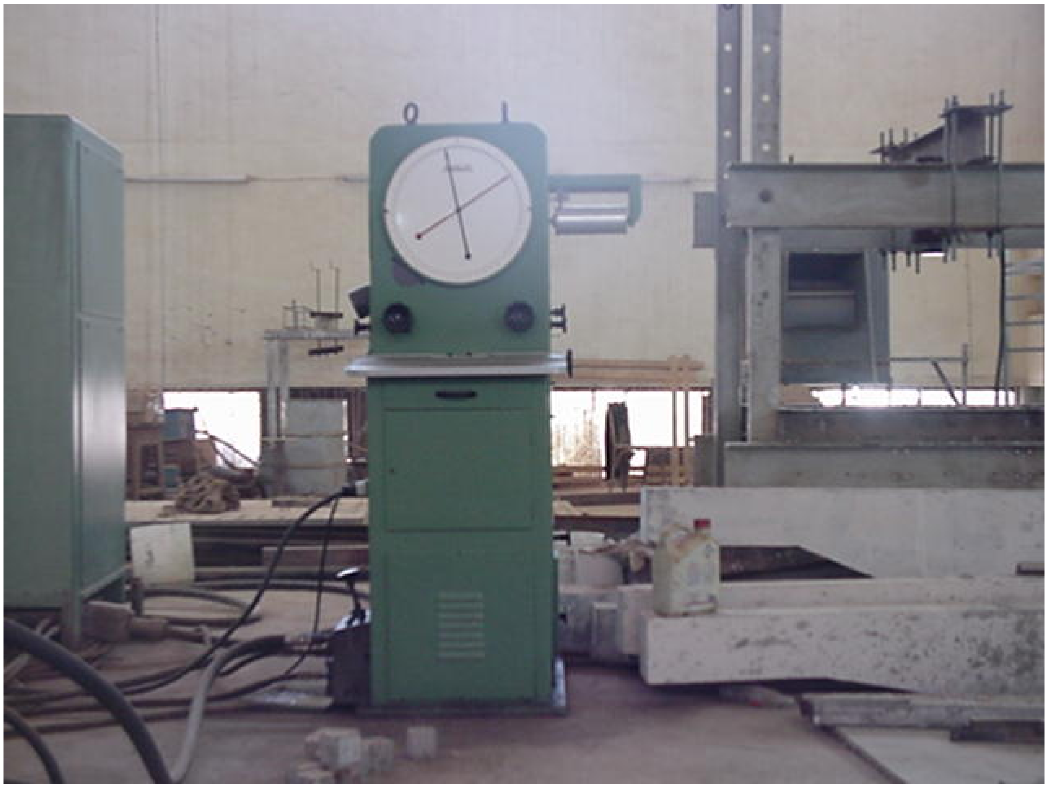
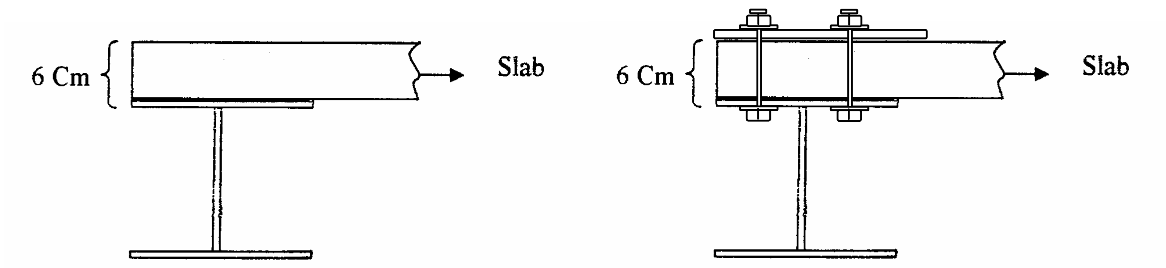


Fig 6.13 View of Testing Frame

6.5.2 Boundary conditions:

The support condition for the tested slabs were, simply supported edges, fixed support.

The simply support edge boundary conditions was attained by rested edges of slab free on the of top flange testing frame to allow rotation to the slab but not to roll away as shown in fig. (6.13)



Simply Supported Fixed Supported

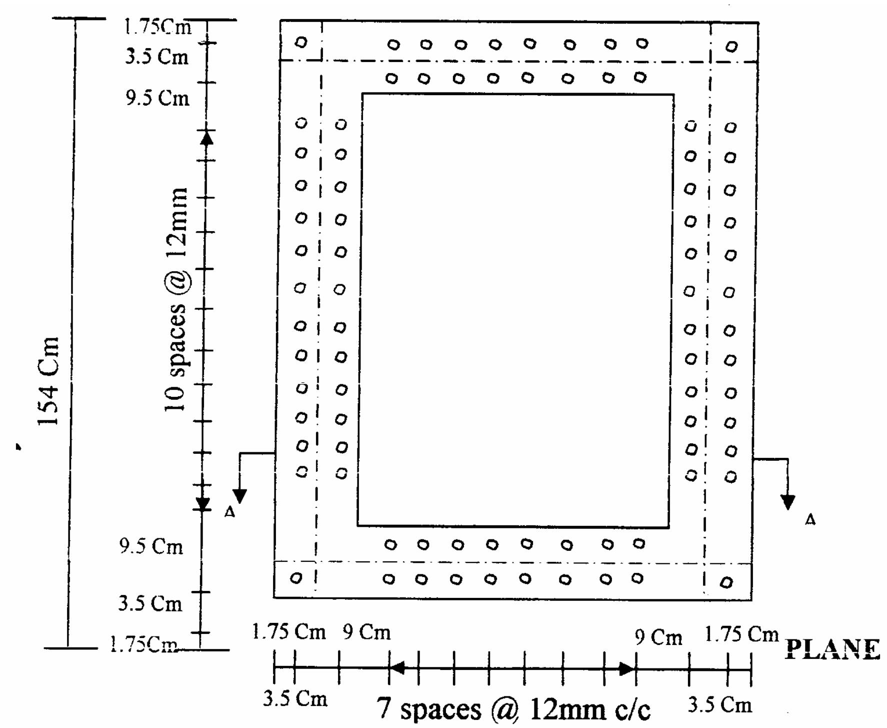


Fig. (6.14) Testing Frame Dimension

The fixed support was achieved by fixed slabs to top of testing frame using 10mm blots spaced 12mm center to center . to increase the fixing, steel plate was introduced at the top of slabs. As shown in Fig. (6.15).

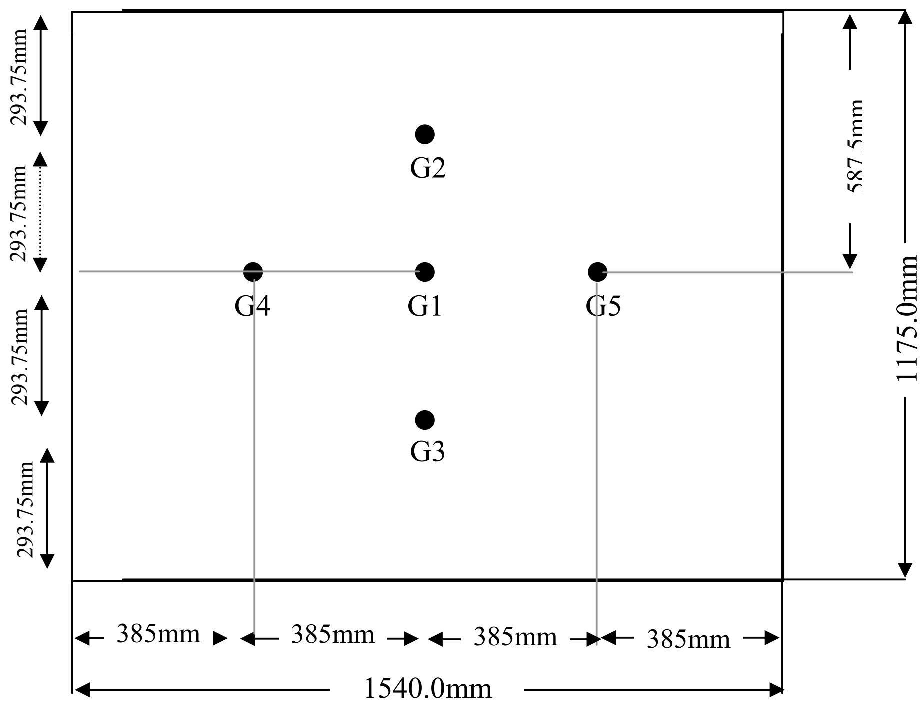


Fig. (6.15) Location of Deflection Measuring For Slabs (A,B)

6.5.3 Application of concentrated Load & Procedure of Testing:

The test model was taken to its position to the loading frame using electrical crane of capacity 50 ton. The slab was free in simply supported slab. In the case of fixed support the slabs were fixed by bolts to testing frame to prevent rotation and horizontal movement of slab during the test.

The model was painted with white lime Water solution in order that the cracks were clearly observed during the test. The locations of deflection gauge on bottom of slabs were marked; mechanical dial gauges of 50mm travel length and an accuracy of 0.01mm were installed for measuring test model. Fig. (6.15) and Fig. (6.16) shows the position of deflection dial guage for the test models.

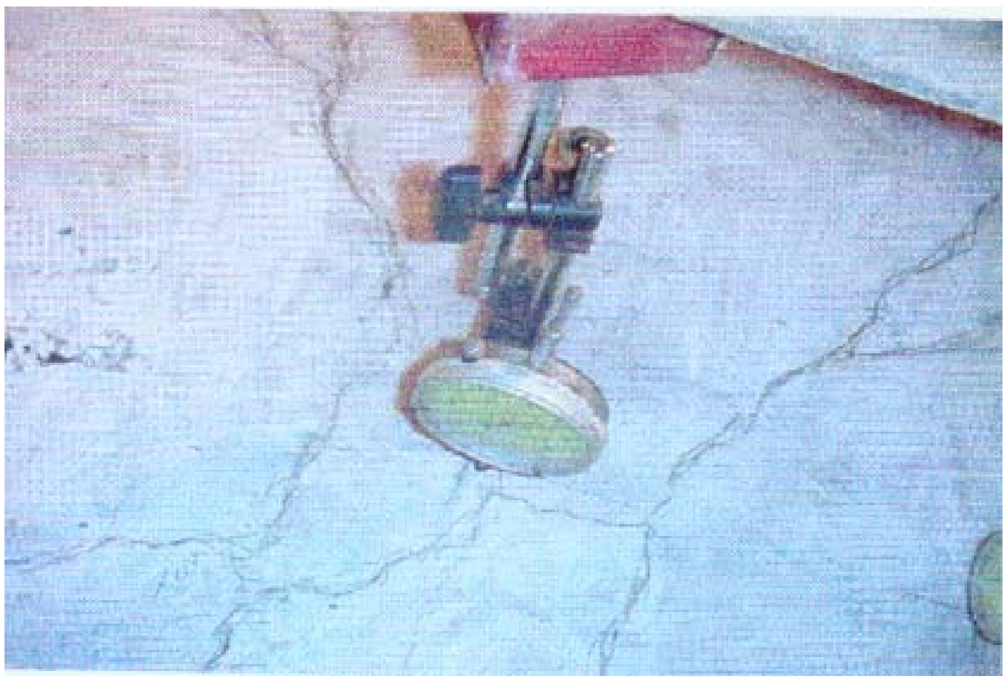


Fig. (6.16) location of deflection measuring points for slabs (A^1 & A^2)

Concentrated load is subjected at the center of slabs by means of manual Jack its capacity was 50 tons. The load was measured by approving ring of capacity 10 tons with a dial guage at its center. A cylindrical steel plate of diameter 8cm was used to transmit the load from the Jack to the slab Fig. (6.12), the diameter of cylinder chosen did not affect the collapse pattern but prevented local crushing failure.

Zero reading of the dial gauges, a proving ring were noted down. The load was noted down. The load was then applied gradually by manual Jack and Load readings were taken from the proving ring. After each loading, the readings of deflections were recorded. The procedure was continued until cracks were visible and the load at which the cracks started was noted. More loading was then applied until the propagation of the cracks was complete and the yield line crack patterns were clearly exhibited. This stage was accompanied by excessive deflections as was clearly indicated by the continuous rotations of the dial gauges, and then the failure load was recorded.

6.6 Experimental Results for Slab:

6.6.1 : Slab A1

Fixed long span
Isotropic reinforcement
Cast :
Tested :

Load Kn	**Deflection mm**					**Remarks**
	G1	G2	G3	G4	G5	
0.00	0.00	0.00	0.00	0.00	0.00	
5.00	0.98	0.41	0.39	0.39	0.42	
10.0	1.44	0.69	0.73	0.76	0.78	
15.0	1.84	0.9	0.87	0.98	1	
20.0	2.42	1.3	1.21	1.4	1.6	First visible crack Pv
25.0	3.52	1.7	1.79	1.8	1.9	
30.0	4.89	2.94	3.1	3.7	3.2	
35.0	5.64	3.3	3.6	3.9	4.0	
40.0	8.94	4.9	4.6	4.1	4.8	
45.0	10.5	5.6	6.3	7.1	6.9	
50.0	11.3	6.1	6.4	7.7	7.9	Ultimate load pex

Table 6.3

6.6.2 : Slab A2

Fixed long span
Orthotropic reinforcement
Cast :
Tested :

Load **Kn**	**Deflection mm**					**R e m a r k s**
	G1	G2	G3	G4	G5	
0.00	0.00	0.00	0.00	0.00	0.00	
5.00	1.24	0.46	0.52	0.51	0.5	
10.0	1.55	0.57	0.53	0.61	0.71	
15.0	2.2	1.34	1.3	1.61	1.41	
20.0	2.53	1.74	1.8	1.82	1.87	First visible crack Pv
25.0	3.35	2.1	2.3	2.83	2.43	
30.0	5.51	3.28.	3.0	3.5	3.42	
35.0	6.93	3.8	4.1	4.3	4.6	
40.0	9.39	5.3	6.0	6.3	6.6	
45.0	11.01	7.2	7.1	8.1	8.3	Ultimate load Pex

Table 6.4

6.6.3 Slab B1

Fixed short span
Isotropic reinforcement
Cast :
Tested :

Load Kn	**Deflection** mm					**R e m a r k s**
	G1	G2	G3	G4	G5	
0.00	0.00	0.00	0.00	0.00	0.00	
5.00	1.43	0.5	0.42	0.47	0.45	
10.0	2.04	1.06	0.81	0.95	0.82	
15.0	2.88	2.0	1.39	1.65	1.52	
20.0	4.7	3.05	3.0	3.6	3.2	First visible crack Pv
25.0	5.83	3.7	3.1	4.0	3.9	
30.0	7.48	3.9	3.3	4.9	3.25	
35.0	10.32	5.0	4.9	6.0	5.8	
40.0	12. 92	7.3	7.6	8.1	7.91	
45.0	13.1	7.2	7.6	8.3	8.1	Ultimate load pex

Table 6.5

6.6.4 Slab B2

Fixed short span
Orthotropic reinforcement
Cast :
Tested :

Load Kn	**Deflection mm**					**Remarks**
	G1	G2	G3	G4	G5	
0.00	0.00	0.00	0.00	0.00	0.00	
5.00	1.5	0.49	0.46	0.51	0.51	
10.0	2.91	1.2	1.12	1.3	1.35	
15.0	2.87	2.21	2.3	1.7	1.56	
20.0	4.16	2.3	2.5	2.7	2.8	First visible crack Pv
25.0	6.0	3.91	4.1	3.8	4.1	
30.0	7.51	4.01	4.7	5.11	4.9	
35.0	11.1	5.31	5.22	6.1	5.79	
37.0	19.1	5.3	6.0	6.2	5.9	Ultimate load pex

Table 6.6

Chapter Seven

Discussion and Analysis of Results:

7.1 Introduction:

The purpose of this chapter is to discuss and compare between the experimental and analytical solutions, the slabs test were analyzed based on the following:

1- Observation of the cracks development.

2- Crack patterns, sketches of crack patterns which are assumed by yield line theory and compared with experimental crack pattern.

3- Deflection data, load deflection curves, comparison between istropic and orthotropic deflection at the same concentrated loads.

4- Failure load, comparison between experimental and theoretical failure load for different slabs.

5- Modes of failure.

7.2 Observation of crack development:

The assumed and observed yield line crack pattern of the slabs is shown in (Fig (7.1-7.4)) for all slabs.

For slab A1 and A2 the first crack appeared at the bottom surface at the center of slab, under concentrated load of 20 kN. Then the crack increased in width and continued to prograte to the corners of supports and crack at the top surface appeared at 35 kN for istropic slab A1, for the orthotropic slab A2 the load is 30 kN.

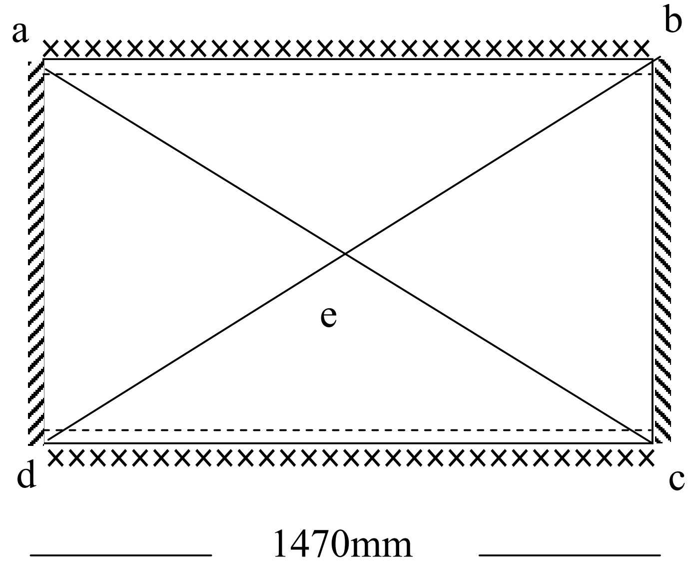
For slab B1, B2 the first crack appeared at the bottom surface at the center of slab under concentrate load of 20 kN, then crack increased in width.

7.3 Yield line and Crack Patterns:

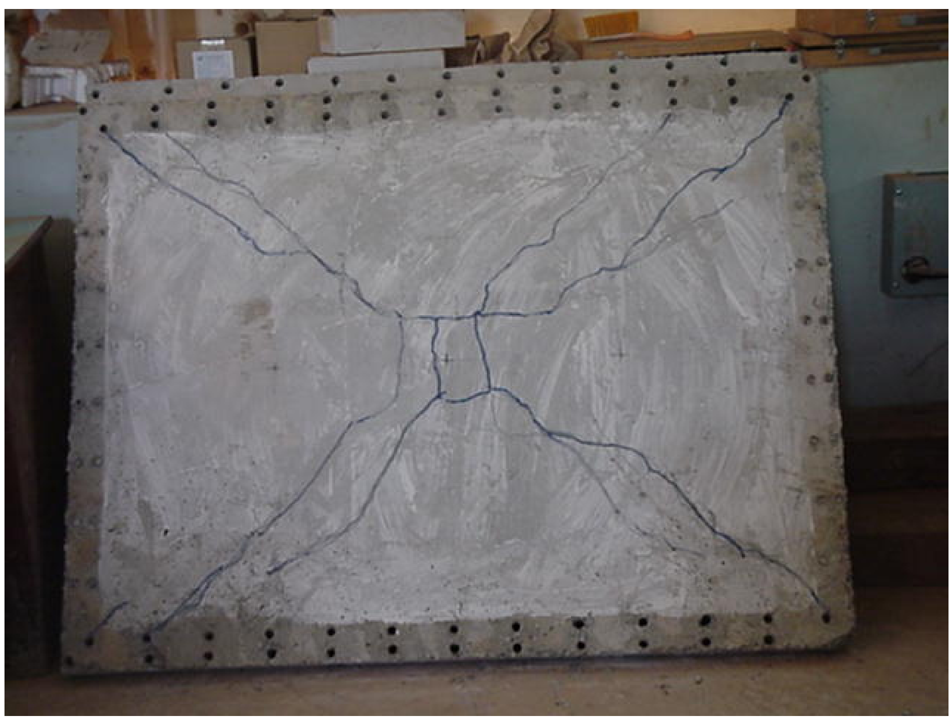
As shown in figure (7.1-7.4) comparison between predicted and experimental yield line patterns is made.

For all slabs (Group (A) and Group (B)) under concentrated load applied at center similar yield line patterns for predicted and experimental yield line pattern were observed at bottom surface (The positive yield line) the negative yield line was observed at the top surface slab but not extended along fixed span.

This disagreement due to the fact that the steel did not yet yield at the top of slab because redistribution of moment have not yet taken place from mid span to supports, so that the second mechanism of support is not yet complete.



(a)



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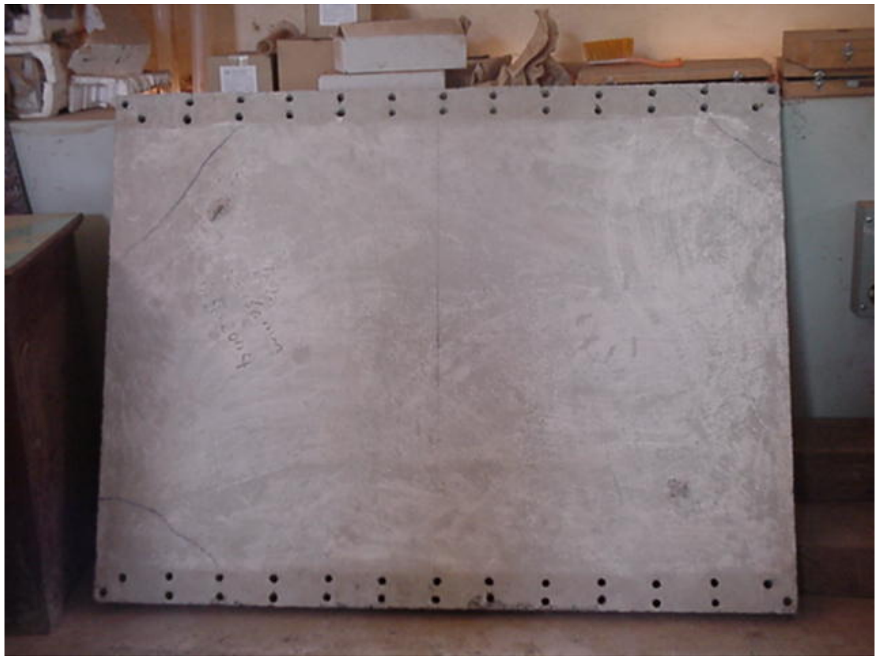


(c)

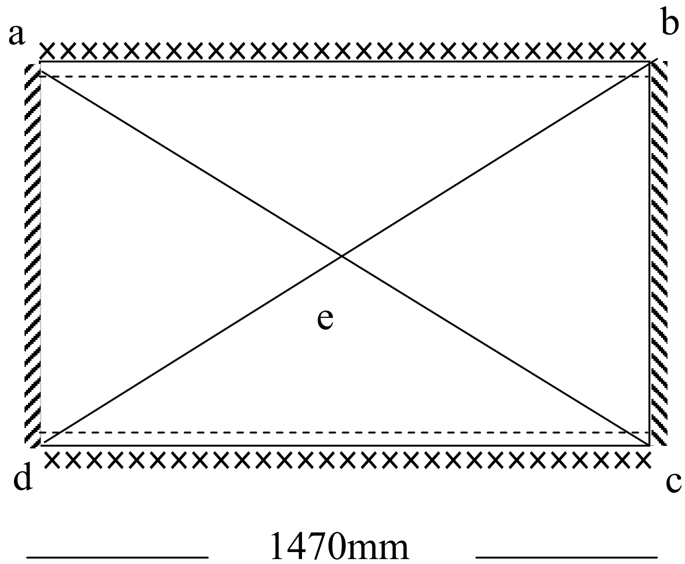
Fig (7.1) Concentrated loaded slab – fixed supported along the long span and simply supported along the short span

(Istropic reinforcement) A1

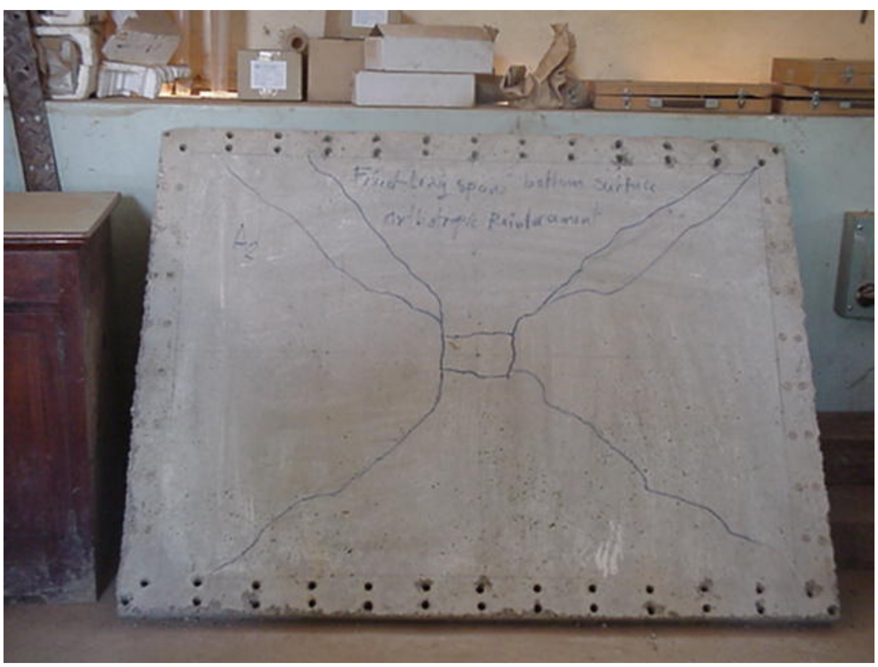
(a) Theoretical yield line pattern

(b) Actual crack pattern on bottom surface.

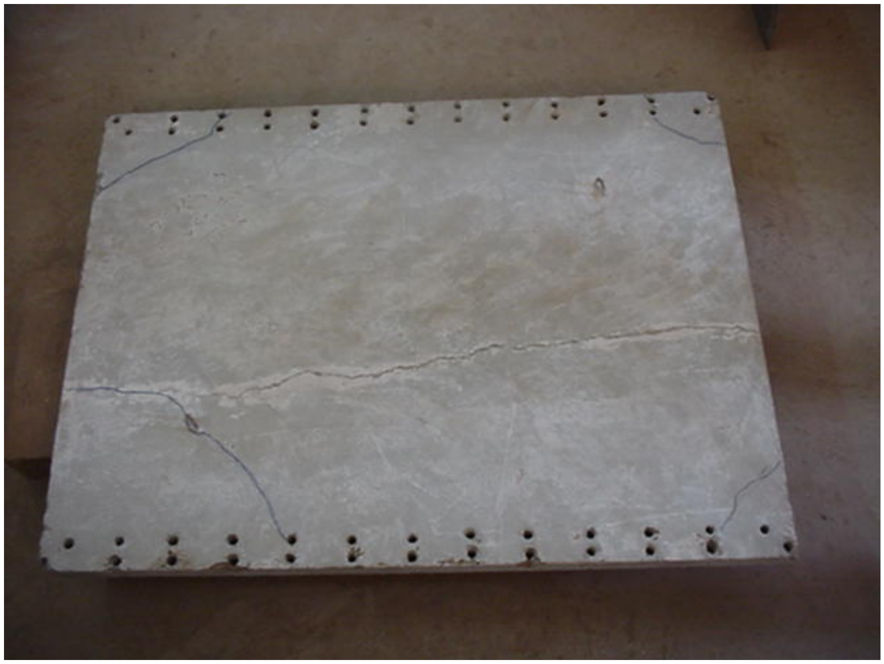
(c) Actual crack pattern on top surface.



(a)



(b)



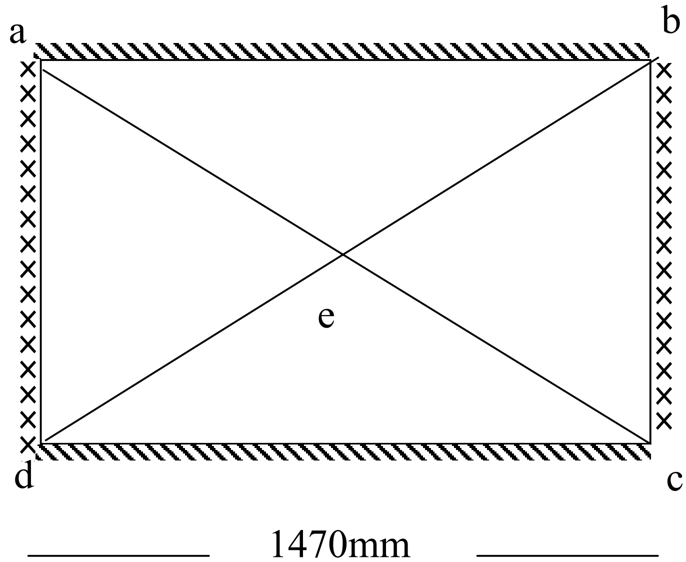
(c)

Fig (7.2) Concentrated loaded slab – fixed supported along long span and simply supported along short span (Orthotropic reinforcement) A2

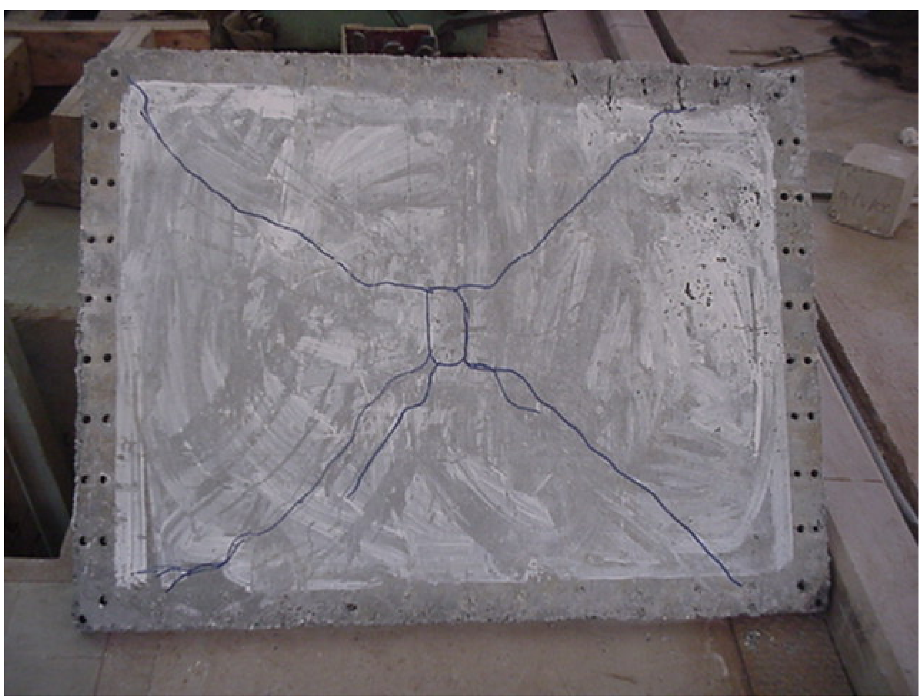
(a) Theoretical yield line pattern

(b) Actual crack pattern on bottom surface.

(c) Actual crack pattern on top surface.



(a)



(b)

s

(c)

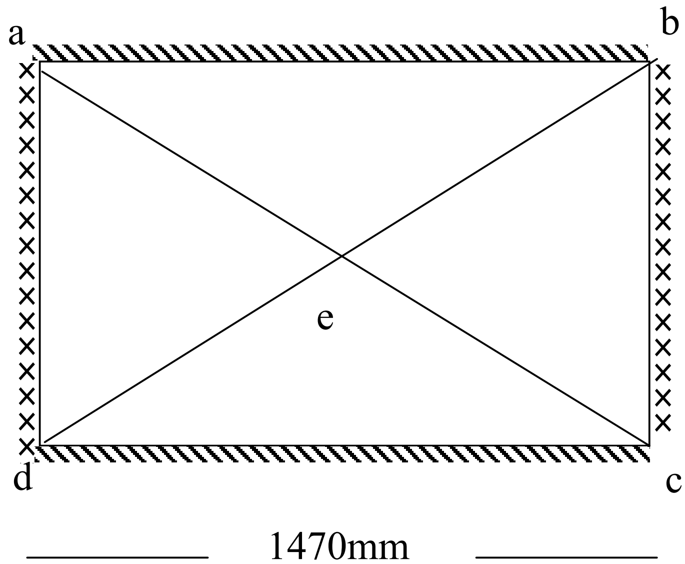
Fig (7.3) Concentrated loaded slab – fixed supported along short span and simply supported along long span

(Istropic reinforcement) B1

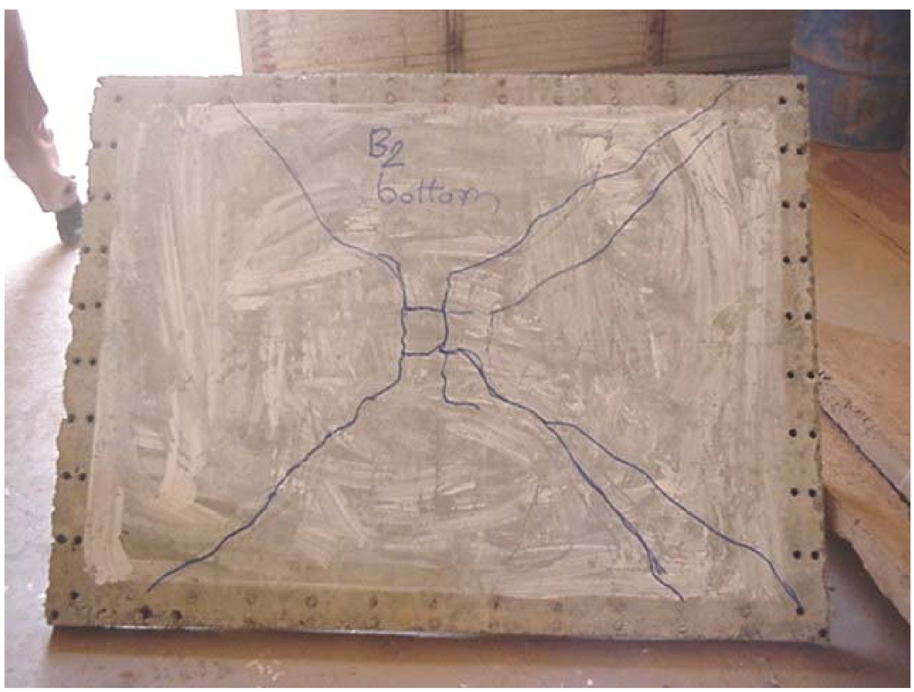
(a) Theoretical yield line pattern

(b) Actual crack pattern on bottom surface.

(c) Actual crack pattern on top surface.



(a)



(b)

(c)

Fig (7.4) Concentrated loaded slab – fixed supported along short span and simply supported along long span (Orthotropic reinforcement) B2

(a) Theoretical yield line pattern

(b) Actual crack pattern on bottom surface.

(c) Actual crack pattern on top surface.

7.4 Ultimate Load Comparison:

A comparison between the theoretical and experimental results of slabs is shown in table (7.1), the following points are observed.

1. For (Group A1) the ratio between experimental and theoretical results was (1.32) for slab A1 and was (1.36) for slab A2.

2- For (Group B) the ratio between experimental and theoretical result was (1.25) for slab B1 it was (1.31) and for slab B2.

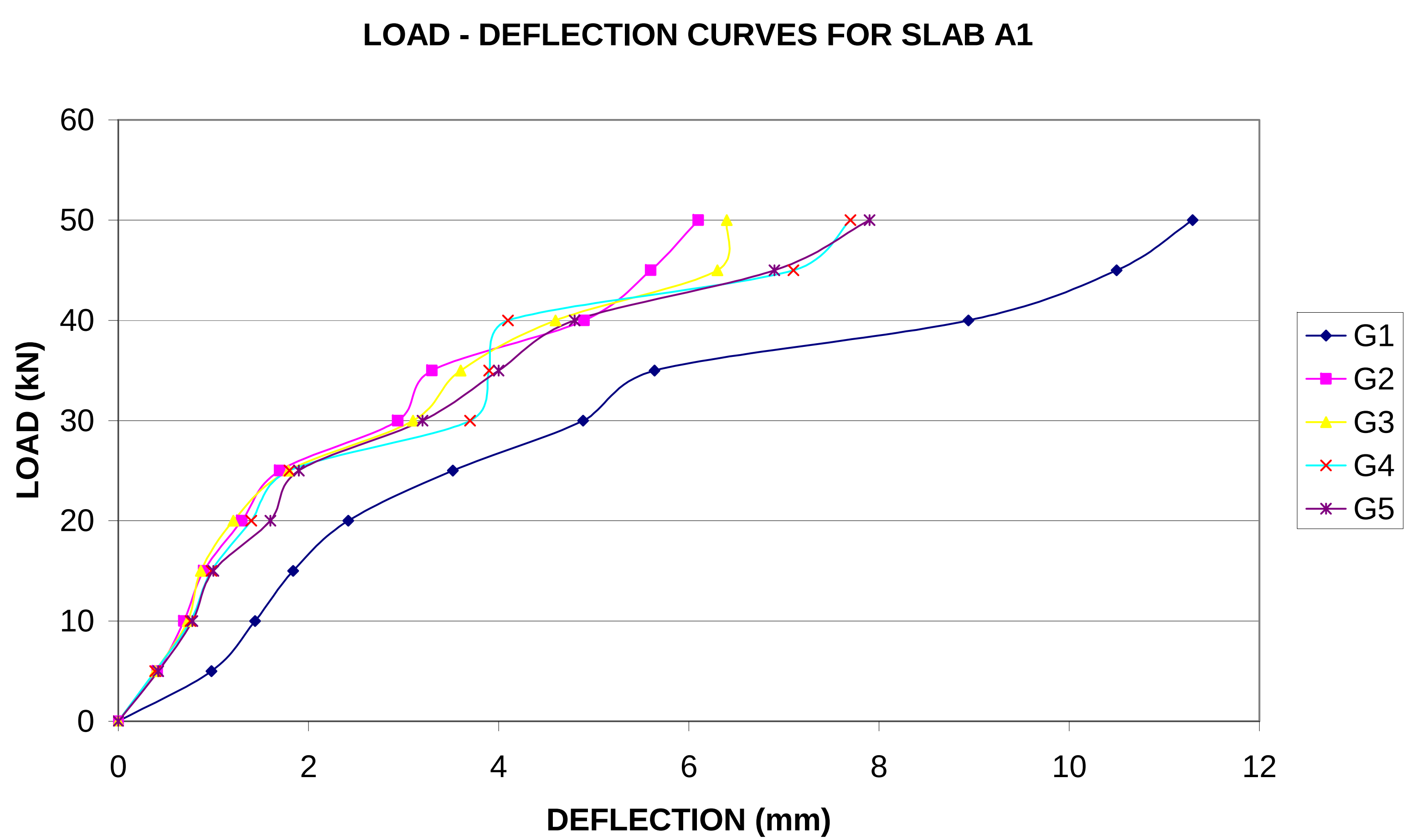
3- For slab A1 istropic reinforcement the ratio between first cracking load to the ultimate load was (0.4) and for slab A2 orthotorpically reinforcement it was (0.44).

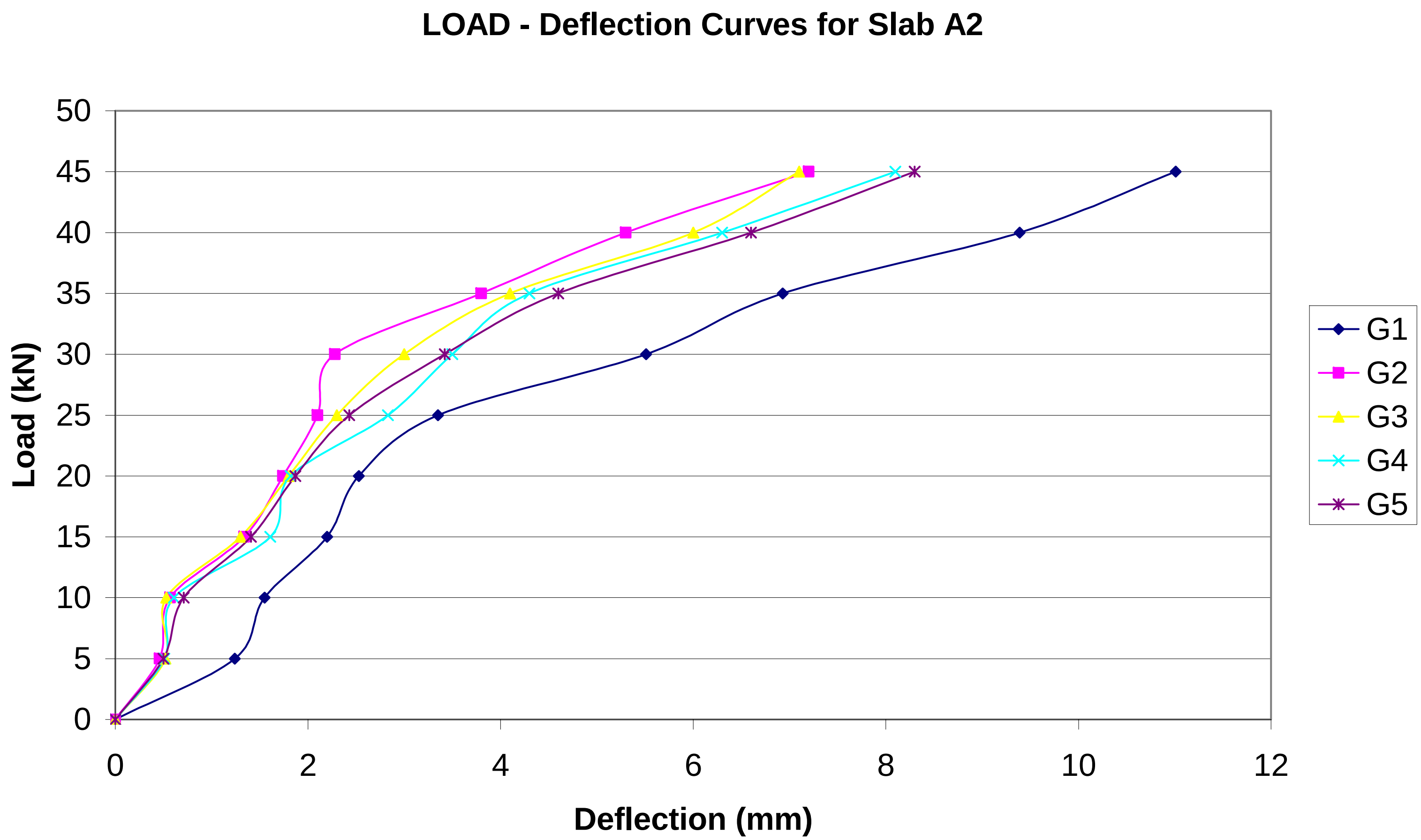
4- For slab B1 isotropic reinforcement the ratio between first cracking load to ultimate load was (0.5) and for slab B2 orthotopically reinforced slab it was (0.54).

7.5 Deflection:

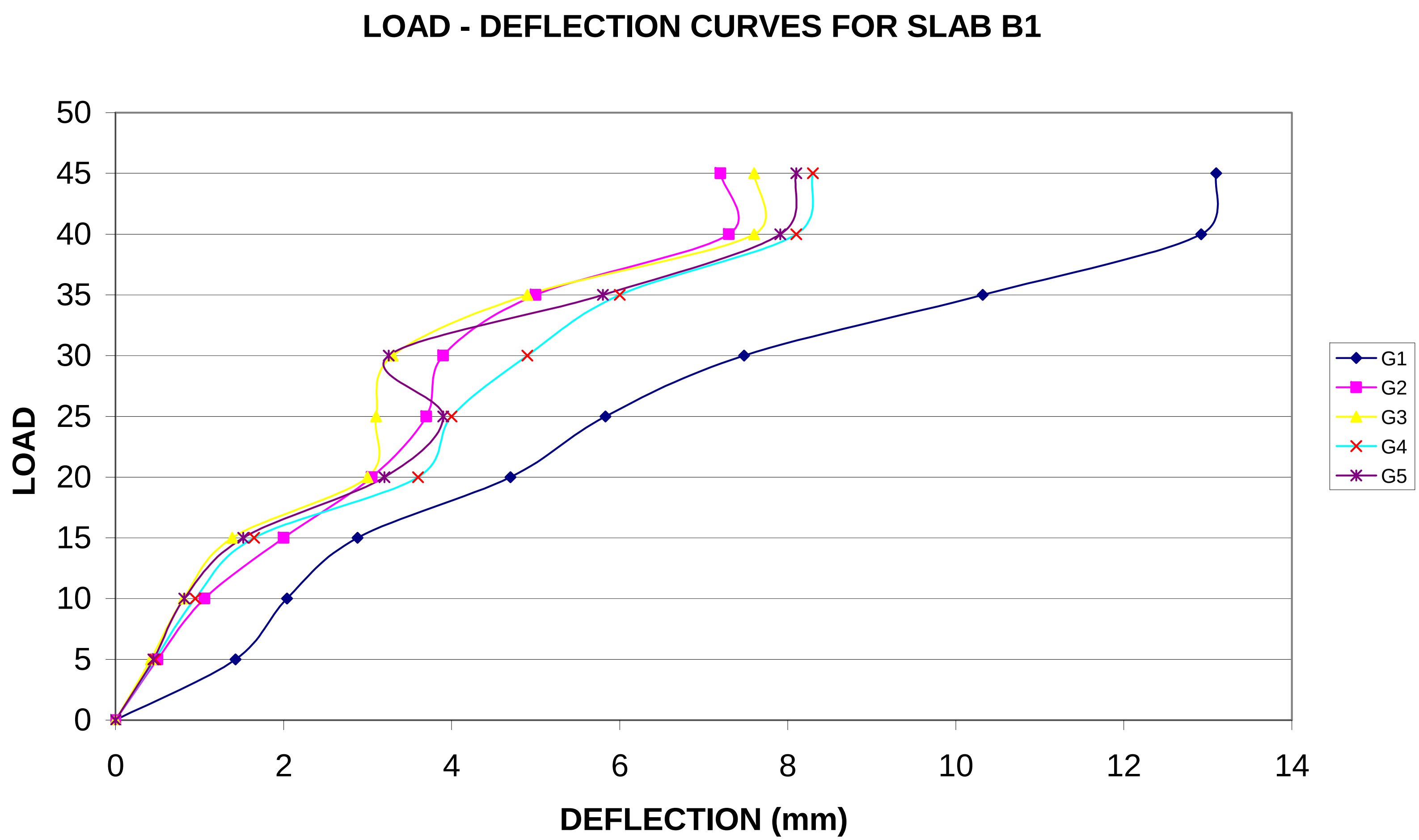
A typical load affection curve obtained from experimental results can be seen in Fig (7.5-7.8).

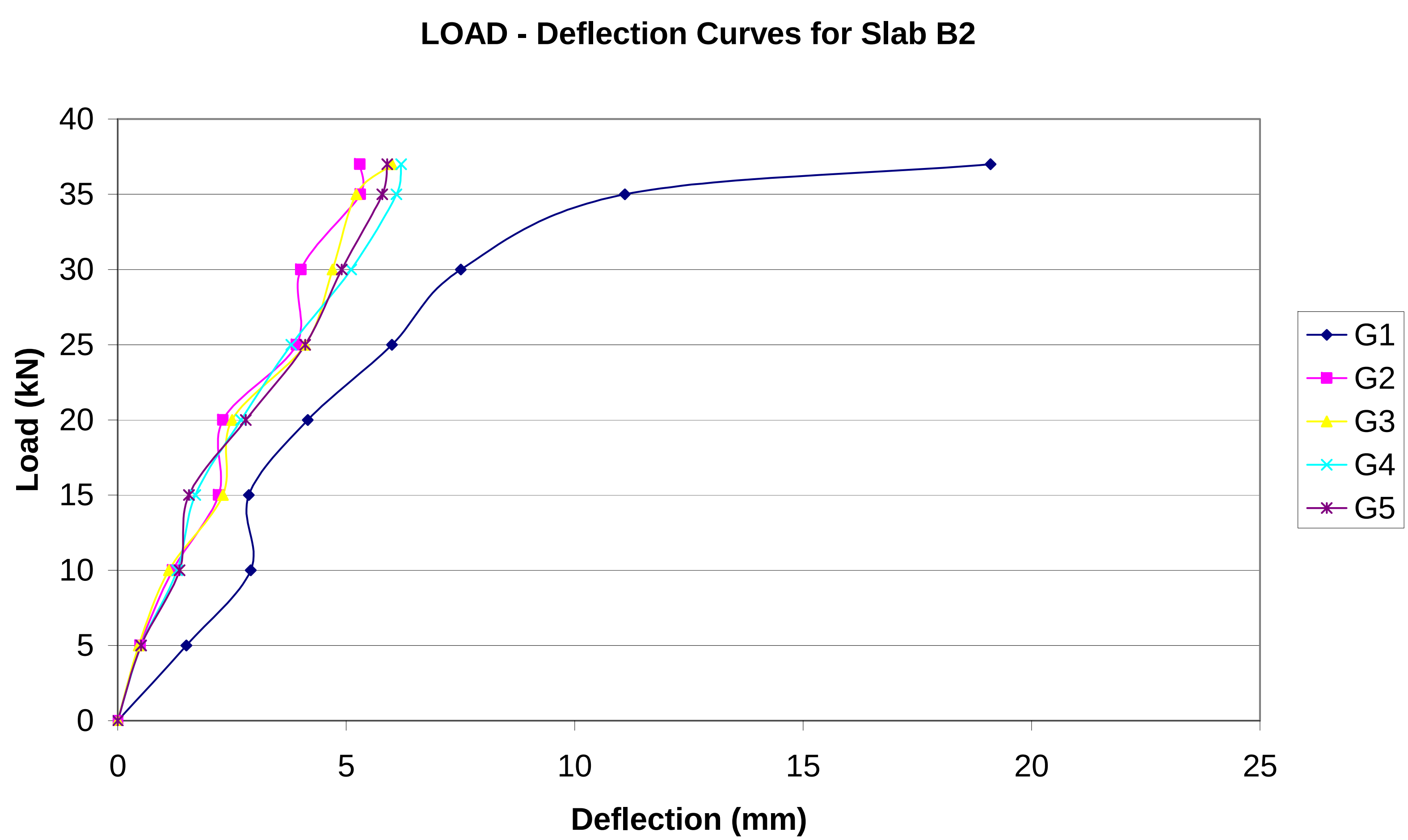
Comparing the valves of the maximum deflections (at midspan) for group (A) and group (B) under the same concentrated loads, it is observed that the deflections of the latter cases are always greater than those for the former. The increase in deflection is due to the fact that for the orthotropic slabs the amount of reinforcement has been reduced appreciably resulting in the reduction of the flexural rigidity of slab as shown in table (6.3-6.6)


LOAD - DEFLECTION CURVES FOR SLAB A1
LOAD (kN)
0
10
20
30
40
50
60
0
2
4
6
8
10
12
DEFLECTION (mm)
G1
G2
G3
G4
G5

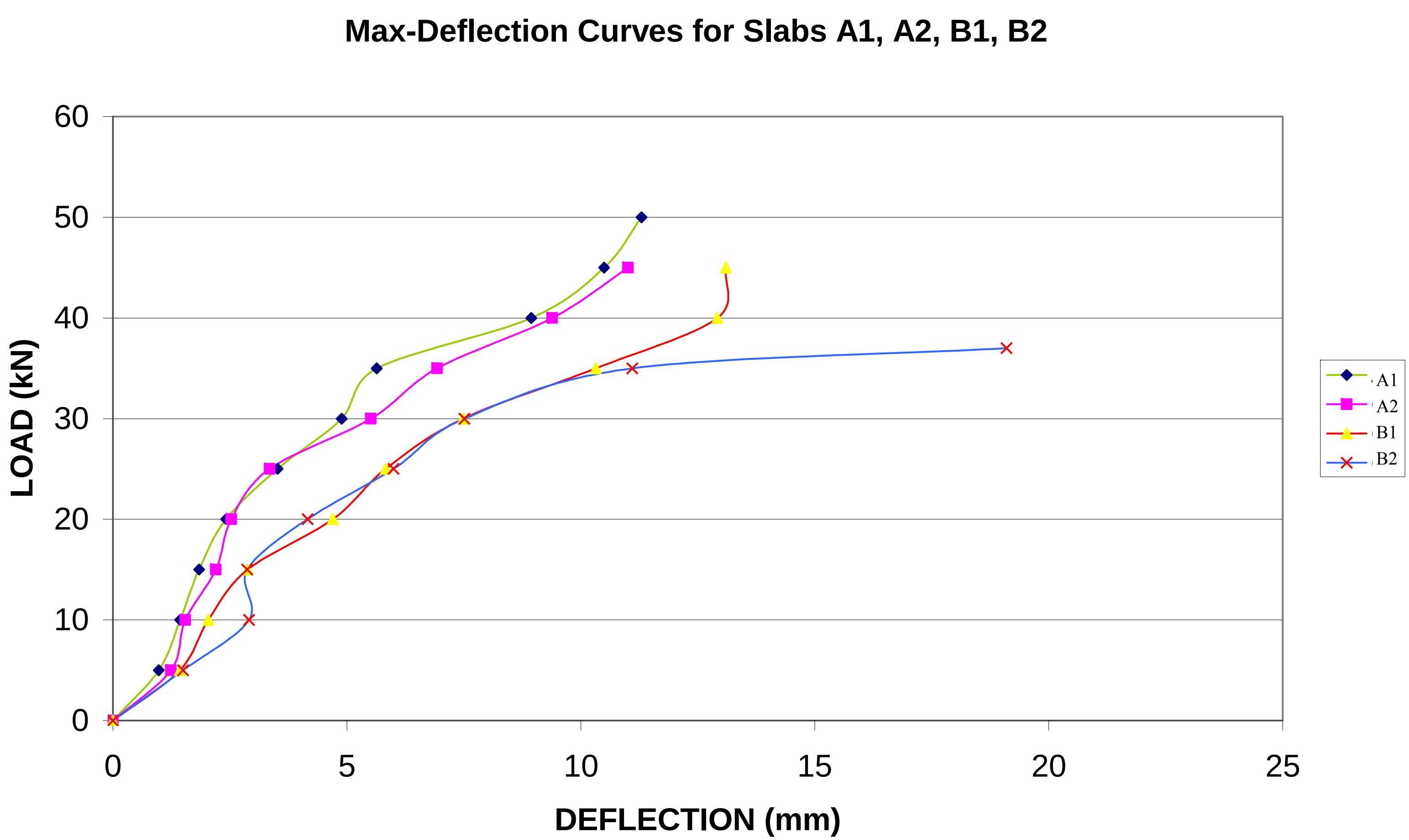


LOAD - Deflection Curves for Slab A2
Load (kN)
0
5
10
15
20
25
30
35
40
45
50
0
2
4
6
8
10
12
Deflection (mm)
G1
G2
G3
G4
G5


LOAD - DEFLECTION CURVES FOR SLAB B1
LOAD
50
45
40
35
30
25
20
15
10
5
0
0
2
4
6
8
10
12
14
DEFLECTION (mm)
G1
G2
G3
G4
G5



LOAD - Deflection Curves for Slab B2
Load (kN)
40
35
30
25
20
15
10
5
0
0
5
10
15
20
25
Deflection (mm)
G1
G2
G3
G4
G5



Max-Deflection Curves for Slabs A1, A2, B1, B2
LOAD (kN)
0
10
20
30
40
50
60
0
5
10
15
20
25
DEFLECTION (mm)
A1
A2
B1
B2

Table (8.1) Test Variable and Comparison of Results

Slabs Group	Slabs No. (Mark)	Dimension (Lx*Ly*h) cm	Type of load	Support Condition	Type of Reinforcement	f_{cu} N/mm^2	F_y N/mm^2	P_V (kN)	P_{exp} (kN)	P^{theo} (kN)	P_{exp}/P_{theo}	P_v/P_{exp}	Deflection at Failure Load (mm)	Failure Mode
1	A1	154 × 117.5 × 6	CON.	F/L	Iso	49.7	586	20	50	37.63	1.32	0.4	11.3	Steel Yielding
	A2	154 × 117.5 × 6	CON.	F/L	Ortho	48.2	386	20	45	32.91	1.36	0.44	10.3	Steel Yielding
2	B1	154 × 117.5 × 6	CON.	F/S	Iso	49.8	386	20	40	31.81	1.25	0.5	12.92	Steel Yielding
	B2	145 × 117.5 × 6	CON.	F/S	Ortho	44.3	386	20	37	28.1	1.31	0.54	9.1	Steel Yielding

Notes:

F/L: Fixed along long span

F/S: Fixed along short span

F_{cu}: Compressive strength of concrete

P_{theo}: theoretical Failure Load

Fy.:Tensile Stress of Reinforcement

CON.: concentrated Load

Iso.: Isotropic Reinforcement

Ortho: Orthotropic Reinforcement

Pv: First Visible Crack

Chapter Eight

Conclusion & Recommendations

1- It is apparent from table (7.1) that the difference between the experimental ultimate load and theory ranged from [36%-25%] this is satisfactory and lies on on the safe side.

2- Despite of the geometrical symmetry of the four tested slabs, the difference in both fixing conditions and steel reinforcement distribution provided noticeable difference in the values of P_{exp} and $P_{theo.}$ It is then required to consider the fixing conditions and steel density in the design of similar reinforced concrete slabs.

3- From tables (6.3-6.6) and as expected it is found that the major deflection is at point G1 where as the deflection values of points G2, G3, G4, G5, are approximately equal because of the equal distances from the point of loading so it is recommended to determine the critical zones at which one should closely monitor the deflection that could take place without visible cracks. Deflections increased considerably after cracking.

4- It is also obvious from table (6.3-6.6) visible cracks are dangerous signs of failure so we should carefully notice and follow up crack to make sure that is does not take place as result of expressive loading.

5- It is found that the cracks resulting from excessive loading as shown in the photos are typical to the mode failure at the lower parts of the slabs but are not at the upper parts as it should be parallel to the fixed edges. This change in failure behavior is due to the insufficient fixing of the edges which gave it allowance to rotate and act as partially simple supports.

To avoid this mode we can cast a edge Beam along fixed span to prevent any partially movement

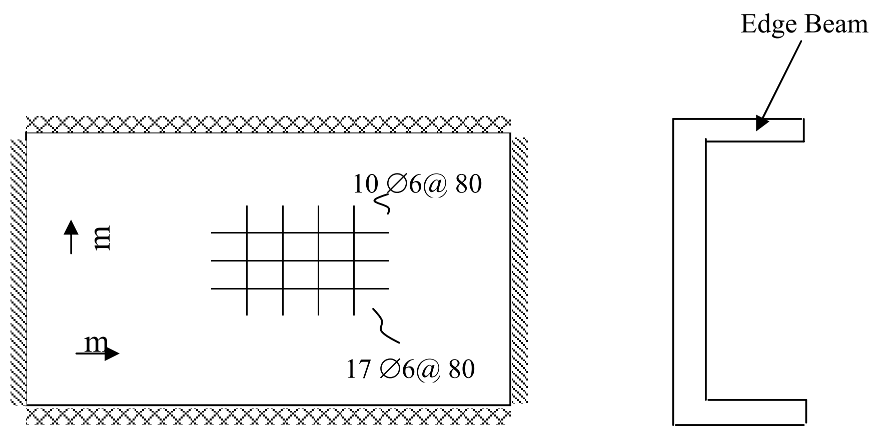


Fig. 8.1

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Appendices

Appendix (1)

1.1 Testing of cement

Consistency and initial and final setting

Type of cement : Ordinary port land cement (marine)

Weight of cement : 400g

Weight of water : 28g

Initial setting time : 2:52 hour (Not less than 15minute)

Final selling time : 3:27 hour (Not more than 10hour)

Consistency : 28%

Strength of cement

1.2 Prism compression test

Weight of sand : 1350g

Weight of Cement : 450g

Weight of Water : 225g

Number of specimen: 3 prism (40mm × 40mm ×16mm)

Specimen No	***Strength N/mm2***	
	2days	28days
1	18.8	51.2
2	19.4	51.9
3	18.1	51.9
A average value	18.8	51.7

Appendix (2)

Seive Analysis

2.1 Seive Analysis test result of fine aggregate

Seive No	**Weight** gram	***Percentage Retrained %***	**Percentage** **Passing** %
5.0	0	0	100%
2.36	14.9	3	97
1.18	119.5	24.2	75.8
0.6	256.2	51.9	48.1
0.3	377	76.4	23.6
0.15	465	94.2	5.8
Total	493.6	100	0

Fine Aggregate classification (zone 2) (B.S812)

Remark : total weight = 2995 kg

2.2 Seive Analysis Test Result of coarse Aggregate

Seive No	**Weight** Gram	*Percentage Retrained %*	**Percentage** **Passing** %
50	0	0	100%
37.5	0	0	100%
20	0	0	100%
14	0	0	100%
10	305	10.3	89.7%
5	2585	87.5	12.5%
Total	2955	100	0

10 mm to 5 mm (B.S 882)

Remark: Total weight = 2955 kg

Appendix (3)

Tensile test of steel

3 Spaciemen

3.1 Specimen NO (1)

Nominal Diameter = 6 mm

Effective Diameter = 5.46 mm

Length = 600mm

Area $= \pi \times \frac{5.46^2}{4} = 23.41$ mm

Load (Ton)	**Elongation** **ΔL**	**Load (N)**	**Stress** **N/mm²**	Strain $\varepsilon = \frac{\Delta L}{L}$
0.0	0	0.0	0.0	0.0
0.2	0.07	2000	85.0	$1.17\times1O^{-4}$
0.4	0.13	4000	173	2.17×10^{-4}
0.6	0.19	6000	256	3.17×10^{-4}
0.8	0.24	8000	342	4.0×10^{-4}
(Y) 0.92	0.9	9200	393	4.5×10^{-4}
1.04	ultimate load	10400	444	-

Elongation = 53.13%

3.2 Specimen NO (2)

Nominal Diameter = 60 mm

Effective Diameter = 5.41 mm

Length = 600mm

Area $= \pi \times \frac{5.41^2}{4} = 22.99$ mm

Load (Tons)	**Elongation (ΔL)**	**Load (N)**	**Stress N/mm^2**	Strain $\varepsilon = \frac{\Delta L}{L}$
0.0	0.0	0.0	0.0	0.0
0.2	0.04	2000	86.99	0.667×10^{-4}
0.4	0.08	4000	173.99	1.33×10^{-4}
0.6	0.14	6000	260.98	2.33×10^{-4}
0.8	0.20	8000	347.98	3.33×10^{-4}
(Y) 0.92	0.27	9200	400.17	4.5×10^{-4}
1.04	Ultimate load	10400	433.51	

Elongation = 53.13 %

3.3 Specimen NO (3)

Nominal Diameter = 60 mm

Effective Diameter = 5.61 mm

Length = 600mm

Area $= \pi \times \frac{5.61^2}{4} = 24.72$ mm

Load (Tons)	Elongation (ΔL)	Load (N)	Stress N/mm^2	Strain $\varepsilon = \frac{\Delta L}{L}$
0.0	0.00	0.0	0.0	0.0
0.2	0.06	2000	80.9	1×10^{-4}
0.4	0.12	4000	16.8	2×10^{-4}
0.6	0.17	6000	242.72	2.83×10^{-4}
0.8	0.23	8000	323.62	3.83×10^{-4}
(Y) 0.9	0.27	9000	367.08	4.5×10^{-4}
1.02	Ultimate load	10200	412.62	-

Elongation = 62.5 %

Results of the tension Test of the steel Reinforcement

Specimen	**fy** **N/mm^2**	**f ult.** **N/mm^2**	*Cross sectional Area* mm^2	*Elongation %*
1	393	444	23.41	26.1
2	400	452	22.99	25.9
3	364	413	24.72	27
Average value	386	436	23.706	

Date of testing : 24.3.2004

Appendix (4)
Mix Design

The following Data Used in the Mix Design
Cement: ordinary Portland
Coarse Aggregate : Crushed stone, maximum size 3/8" (10mm)
Fine Aggregate : Zone 2 (Seive Analysis)
Slump : 30-60
V.B : 3-6 sec
Characteristic Compressive Strength : 30 N/mm2
The mix design procedure is according to DOE mix design method (The department of the univronment's design of normal concrete mixes)
Steps of DOE mix design procedure:

Step 1:

Determining of free W/C ratio:
Target mean strength = f_c + KS
K 1.64 S = 8
Fm = 30 + 1.64 X 8 = 44 N/mm^2
For W/C = 0.5 and crushed Aggregate, from table (2)
Compressive strength = 47 N/mm2
Fig. (4) (W/C = 0.5, with 47 N/mm2) W/C (=0.53)

Step 2:

Determining the water Content:
From Table (3) , maximum size crushed of Aggregate = 10mm
Slump 30-60
Water content = 230 kg/m^3

Step 3:

$$\text{Cement content (kh/m3)} = \frac{\text{Water Content}}{\text{W/C ration}}$$

$$= \frac{230}{0.53} = 433 \text{ kg/m}^3$$

Step 4:

Determining the aggregate content:
For crushed aggregate, relative density = 2.7
Density of wet concrete = 2400 kg/m^3 Fig. (5)
Density of Aggregate = 2400 – 230 – 433 = 1737 kg/m^3
Figure (6): For (30-60)mm slump (3-6)s
Maximum size = 10 mm
Proportion of fine Aggregate 41%
Fine Aggregate = 0.46 X 1737 = 799 kg/m^3
Course Aggregate = 1737 – 799 = 937 kg/m^3

Quantities of Material for Trial and Mix Design

Quantities	**Cement (kg)**	**Water (kg)**	**Fine Aggregate (kg)**	**Course Aggregate (kg)**
Per m^3	435	230	750	960
Per trial mix of $0.006m^3$	2.6	1.4	4.5	5.8

Appendix 5

Compressive Strength Test Results

Type of slump	Slump	No of Specimen	Weight	Load (kN)	Compressive Strength	No. of Spcimen Speed	Weight	Load (kN0	Compressive Stress
Trial mix design date of casting 3,4,2004	55	1	2.555	230	23	1	2560	447	44.7
		2	2.600	2.40	24	2	2590	421	42.1
		3	2.575	2.35	23.5	3	2660	425	42.5
		Average			23.5	Average			43.1
A1	55	1	2.385	310	31	1	2530	490	49
		2	2.345	305	30.5	2	2550	490	49
		3	2.360	295	29.5	3	2550	515	51.1
		Average			30.3	Average			49.7
A2	50	1	2.400	285	28.5	1	2530	470	47
		2	2.340	300	30.28	2	2525	480	48
		3	2.350	280	28.8	3	2540	496	49.6
		Average			37	Average			48.2
B1	60	1	2540	370	36	1	2505	522	52.2
		2	2520	360	39	2	2490	472	47.2
		3	2545	390	37.3	3	2530	500	50.0
		Average			21	Average			49.8
B2	50	1	2530	210	23.5	1	2535	440	44
		2	2555	235	23.5	2	2540	440	44
		3	2534	235	22.7	3	2550	450	45
		Average			29.32	Average			44.3

- Area = 100 Cm^2

Appendix (6)

Mix Design

The following Data Used in the Mix Design

Cement: ordinary Portland

Coarse Aggregate : Crashed stone, maximum size 3/8″ (10mm)

Fine Aggregate : Zone 2 (Seive Analysis)

Slump : 30-60

V.B : 3-6 sec

Characteristic Compressive Strength : 30 N/mm2

The mix design procedure is according to DOE mix design method (The department of the univronment's design of normal concrete mixes)

Steps of DOE mix design procedure:

Step 1:

Determining of free W/C ratio:

Target mean strength = f_c + K5

K 1.64 S = 8

Fm = 30 + 1.64 X 8 = 44 N/mm^2

For W/C = 0.5 and crushed Aggregate, from table (2)

Compressive strength = 47 N/mm2

Fig. (4) (W/C = 0.5, with 47 N/mm2) W/C (=0.53)

Step 2:

Determining the water Content:

From Table (3) , maximum size crushed of Aggregate = 10mm

Slump 30-60

Water content = 230 kg/m^3

Step 3:

Cement content (kh/m3) = $\frac{\text{Water Content}}{\text{W/C ration}}$

$= \frac{230}{0.53}$ = 433 kg/m^3

Step 4:

Determining the aggregate content:

For crushed aggregate, relative density = 2.7

Density of wet concrete = 2400 kg/m^3 Fig. (5)

Density of Aggregate = 2400 – 230 – 433 = 1737 kg/m^3

Figure (6): For (30-60)mm slump (3-6)s

Maximum size = 10 mm

Proportion of fine Aggregate 41%

Fine Aggregate = 0.46 X 1737 = 799 kg/m3

Course Aggregate = 1737 – 799 = 937 kg/m3